SPM ADDITIONAL MATHEMATICS KBAT COLLECTION

FAQ

What is this document and what is KBAT?

This document is a collection of KBAT (Kemahiran Berfikir Asas Tinggi) or HOTS (High Order Thinking Skills) questions of the SPM Additional Mathematics subject. Which are widely known as questions that require students to analyse any given information that are given from the question very carefully, and think of the most efficient and suitable way to approach the question.

Are the questions difficult?

The questions are generally deemed as difficult due to the questions being asked are not as straight-forward as most question do, and are not asked within the usual SPM ‘‘pattern’’. Which prevents students for being too over-reliant on past-year papers for practice, but requires them to solve them on-the-spot instead of observing the patterns that the question is being asked.

Where are the questions from?

The questions collected are mostly from textbooks, past-year SPM papers, past-year trial papers, textbooks and reference books. But there are some questions that are obtained from the Internet as well.

Can all questions be solved?

All of the questions in this document are all within the current KSSM SPM syllabus and hence can be solved. (e.g. Integration by parts and complex numbers are not included.)

What are the general ‘rules’ for solving the questions?

There are no rules for solving the questions, there may be multiple ways to obtain the solution, but only one solution exists for all questions (unless stated in the question). However, questions being asked in any given chapter can be solved with what is being taught **in that chapter or prior** and does **not** require anything that is taught after that chapter. (e.g. Calculus are not required in quadratic functions and coordinate geometry, but solution of triangles may be needed for some trigonometric function questions.)

Are all questions in this document difficult?

Not all questions are difficult. Some of them are simple but are asked in an arbitrary way. Although all questions are difficult, some are more difficult than others. Each question's difficulty can be predicted from the marks allocated on each question, ranging from 2 marks (simple) to 8 marks (difficult) in one sub-question.

How much time should I spend on each question?

In general, students should spend 1.5 minutes per mark given for every question. But since these are all KBAT questions, students tend to spend more time on these questions. So it's probably best to use 2 to 3 minutes at most per mark for these questions.

Is the answer scheme available?

Not yet, it'll probably be available sometime in May or June 2022.

Is there a Malay version?

Not yet, but if you want to help and translate these questions from English to Malay, it'll be the very appreciated.

Will the file get updated?

Yes, the file will be updated from time to time if more questions have been found. But it'll probably get updated if there's any typos or font issues in the question as well.

The following formulae may be helpful in answering the questions. The symbols given are the ones commonly used.

Only the ones in blue are given in the SPM examination, the ones in black are not.

ALGEBRA

1 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

2 $a^m \times a^n = a^{m+n}$

3 $a^m \div a^n = a^{m-n}$

4 $(a^m)^n = a^{m \times n}$

5 $a^{-m} = \frac{1}{a^m}$

6 $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

7 $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

8 $\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$

9 $\log_a m + \log_a n = \log_a mn$

10 $\log_a m - \log_a n = \log_a \frac{m}{n}$

11 $\log_a m^n = n \log_a m$

12 $\log_a b = \frac{\log_c b}{\log_c a}$

13 $T_n = a + (n-1)d$

14 $T_n = ar^{n-1}$

15 $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$

16 $S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}, r \neq 1$

17 $S_\infty = \frac{a}{1 - r}, |r| < 1$

GEOMETRY

1 Distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2 Midpoint $(x, y) = \dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}$

3 A point dividing the line segment $(x, y) = \dfrac{nx_1 + mx_2}{m + n}, \dfrac{ny_1 + my_2}{m + n}$

4 For parallel lines , $m_1 = m_2$

5 For perpendicular lines , $m_1 m_2 = -1$

6 Area of triangle $= \dfrac{1}{2} |(x_1 y_2 + x_2 y_3 + x_3 y_1) - (x_2 y_1 + x_3 y_2 + x_1 y_3)|$

7 $|\underline{r}| = \sqrt{x^2 + y^2}$

8 $\hat{\underline{r}} = \dfrac{x\underline{i} + y\underline{j}}{\sqrt{x^2 + y^2}}$

CALCULUS

1 $\dfrac{\mathrm{d}}{\mathrm{d}x}\ ax^n = n\left(ax^{n-1}\right)$

2 $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}t} \times \dfrac{\mathrm{d}t}{\mathrm{d}x}$

3 $y = uv, \dfrac{\mathrm{d}y}{\mathrm{d}x} = v\dfrac{\mathrm{d}u}{\mathrm{d}x} + u\dfrac{\mathrm{d}v}{\mathrm{d}x}$

4 $y = \dfrac{u}{v}, \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

5 $\displaystyle\int ax^n\ \mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$

6 $\displaystyle\int (ax + b)^n\ \mathrm{d}x = \frac{(ax + b)^{n+1}}{a(n+1)} + c$

7 Area under a curve $\displaystyle= \int_a^b y\ \mathrm{d}x$ or $\displaystyle\int_a^b x\ \mathrm{d}y$

8 Volume of revolution $\displaystyle= \int_a^b \pi y^2\ \mathrm{d}x$ or $\displaystyle\int_a^b \pi x^2\ \mathrm{d}y$

STATISTICS

1 $I = \frac{Q_1}{Q_0} \times 100$

2 $\overline{I} = \frac{\sum W_i I_i}{W_i}$

3 $n! = n(n-1)!$

4 ${}^{n}P_r = \frac{n!}{(n-r)!}$

5 Number of circular permutations $= \frac{{}^{n}P_r}{r}$

6 Number of permutations involving identical objects $= \frac{n!}{a!\,b!\,c!\ldots}$

7 ${}^{n}C_r = \frac{n!}{(n-r)!\,r!}$

8 $P(X = r) = {}^{n}C_r\, p^r q^{n-r}$, $p + q = 1$

9 $\mu = np$

10 $\sigma = \sqrt{npq}$

11 $Z = \frac{X - \mu}{\sigma}$

TRIGONOMETRY

1 $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

2 $a^2 = b^2 + c^2 - 2bc\cos A$

3 Area of triangle $= \dfrac{1}{2}ab\sin C = \sqrt{s(s-a)(s-b)(s-c)}, s = \dfrac{a+b+c}{2}$

4 Arc length, $s = r\theta$

5 Area of sector, $A = \dfrac{1}{2}r^2\theta$

6 $\sin^2 A + \cos^2 A = 1$

7 $\tan^2 A + 1 = \sec^2 A$

8 $1 + \cot^2 A = \operatorname{cosec}^2 A$

9 $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

10 $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

11 $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

12 $\sin 2A = 2\sin A\cos A$

13 $$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

14 $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

15 $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}$

16 $\cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}}$

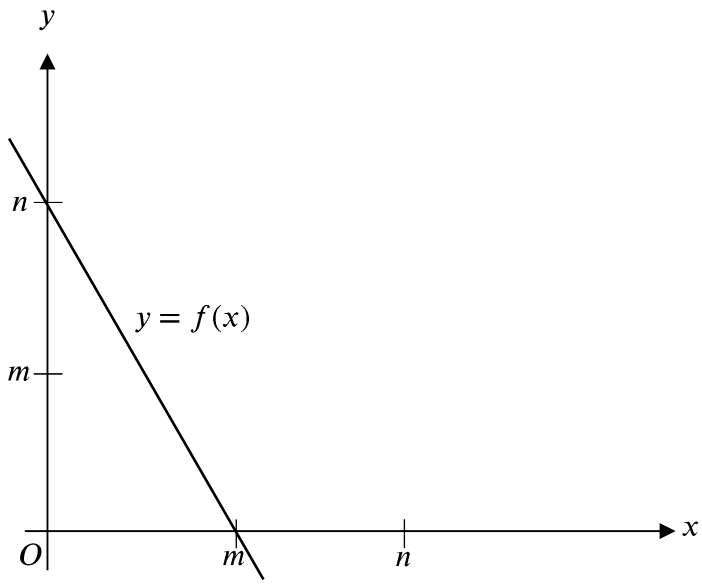
17 $$\begin{aligned}\tan\frac{A}{2} &= \frac{\sin A}{1+\cos A}\\ &= \frac{1-\cos A}{\sin A}\\ &= \pm\sqrt{\frac{1-\cos A}{1+\cos A}}\end{aligned}$$

THE UPPER TAIL PROBABILITY $Q(z)$ FOR THE NORMAL DISTRIBUTION N(0, 1)

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MINUS								
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4961	.4721	.4681	.4641	4	8	12	16	20	24	28	32	36
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247	4	8	12	16	20	24	28	32	36
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859	4	8	12	15	19	23	27	31	35
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483	4	7	11	15	19	22	26	30	34
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121	4	7	11	14	18	22	25	29	32
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2943	.2810	.2776	3	7	10	14	17	20	24	27	31
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451	3	7	10	13	16	19	23	26	29
0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148	3	6	9	12	15	18	21	24	27
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867	3	5	8	11	14	16	19	22	25
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611	3	5	8	10	13	15	18	20	23
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379	2	5	7	9	12	14	16	19	21
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170	2	4	6	8	10	12	14	16	18
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985	2	4	6	7	9	11	13	15	17
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823	2	3	5	6	8	10	11	13	14
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0684	.0681	1	3	4	6	7	8	10	11	13
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559	1	2	4	5	6	7	8	10	11
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455	1	2	3	4	5	6	7	8	9
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367	1	2	3	4	4	5	6	7	8
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294	1	1	2	3	4	4	5	6	6
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233	1	1	2	2	3	4	4	5	5
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183	0	1	1	2	2	3	3	4	4
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143	0	1	1	2	2	2	3	3	4
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110	0	1	1	1	2	2	2	3	3
2.3	.0107	.0104	.0102								0	1	1	1	1	2	2	2	2
				.00990	.00964	.00939	.00914				3	5	8	10	13	15	18	20	23
								.00889	.00866	.00842	2	5	7	9	12	14	16	18	21
2.4	.00820	.00798	.00776	.00755	.00734						2	4	6	8	11	13	15	17	19
						.00714	.00695	.00676	.00657	.00639	2	4	6	7	9	11	13	15	17
2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480	2	3	5	6	8	9	11	12	14
2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357	1	2	3	5	6	7	8	9	10
2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264	1	2	3	4	5	6	7	8	9
2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193	1	1	2	3	4	4	5	6	6
2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139	0	1	1	2	2	3	3	4	4
3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100	0	1	1	2	2	2	3	3	4

FUNCTIONS

1 Diagram below shows part of the graph $y = f(x)$.



(a) Find $f^{-1}(0)$.

(b) Sketch the graph of $y = f^{-1}(x)$.

[2 *marks*]

2 It is given that $f(x) = \dfrac{ax + b}{cx + d}, x \neq -\dfrac{d}{c}$

Find

(a) $f^{-1}(x)$,

(b) the inverse function for each of the following functions,

i) $g(x) = \dfrac{x + 8}{x - 5}, x \neq 5$

ii) $h(x) = \dfrac{2x - 3}{x + 4}, x \neq -4$

(c) the conditions for a, b, c and d such that $f(x) = f^{-1}(x)$ if $c \neq 0$.

[5 *marks*]

3 The price of an item, p, and the quantity, x, of the item sold follow the demand equation $p = 100 - \frac{x}{4}$ for $0 \le x \le 400$. Whereas the cost, C, in RM, to produce x units is given by $C = \frac{\sqrt{x}}{25} + 600$. Assuming all the items produced are sold, calculate

(*a*) the cost, C as a function of price, p,

(*b*) the cost for producing that item if the price for one unit of item is sold at RM 36.

[4 *marks*]

4 The function f is defined as $f(x) = \frac{x-1}{x+1}, x \neq -1$.

(*a*) Find

i) $f^2(x)$.

ii) $f^4(x)$.

(*b*) Hence, determine $f^{26}(x)$.

[5 *marks*]

5 Given $g(x) = \frac{2x-5}{3x-p}, x \neq q$.

(*a*) i) Express $g^{-1}(x)$ in terms of p.

ii) If $g^{n-1}(x) = \frac{px-5}{3x-2}, x \neq \frac{2}{3}$. Determine the value of n.

(*b*) Determine the value of q if $g = g^{-1}$.

[5 *marks*]

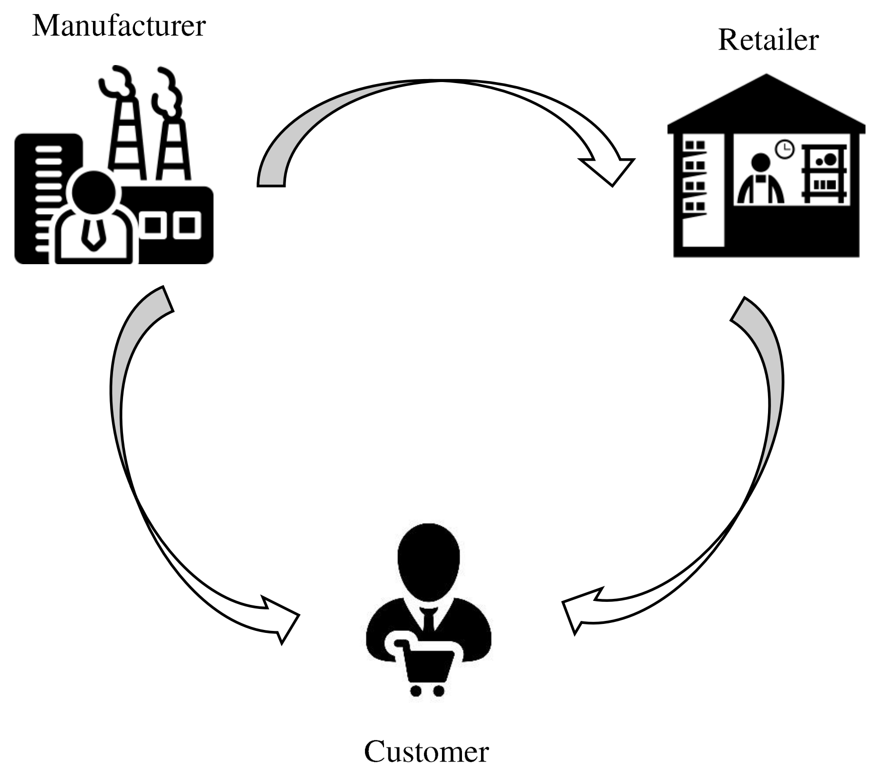
6 Given that $f(x) = x^2 - 11x + 33$ has an inverse if $x \ge p$ or $x \le p$, where p is a constant.

(*a*) Find the value of p.

(*b*) Hence, line $y = a$ is drawn such that a is an integer that is less than $f(p)$. Determine the possible number of solutions for the equation $a = x^2 - 11x + 33$.

[2 *marks*]

7 Diagram below shows a manufacturer, a retailer, and a customer.



The manufacturer produces some items and sell them to the retailer depending on the cost of the item produced. It is given that the price of the item for the retailer, P, in RM, for an item with a cost of RM x, is given by $2x + 10$. The retailer then follows a function and sells the item to the customer. If the customer, however, decides to buy the item directly from the manufacturer, the price will be six times of the item's cost, but is RM 35 cheaper than the price that the customer will get from the retailer.

Based on the information above, find

(*a*) the price of the item, in RM, from the retailer, in terms of x,

(*b*) the price of the item, in RM, from the retailer, as a function of P,

(*c*) the range of values of x, such that the retailer profits from selling the item.

[7 *marks*]

QUADRATIC FUNCTIONS

1 Given that the quadratic equation $2x^2 = ax - b$ has roots α and β such that $\alpha - 2\beta = 0$. Express a in terms of b.

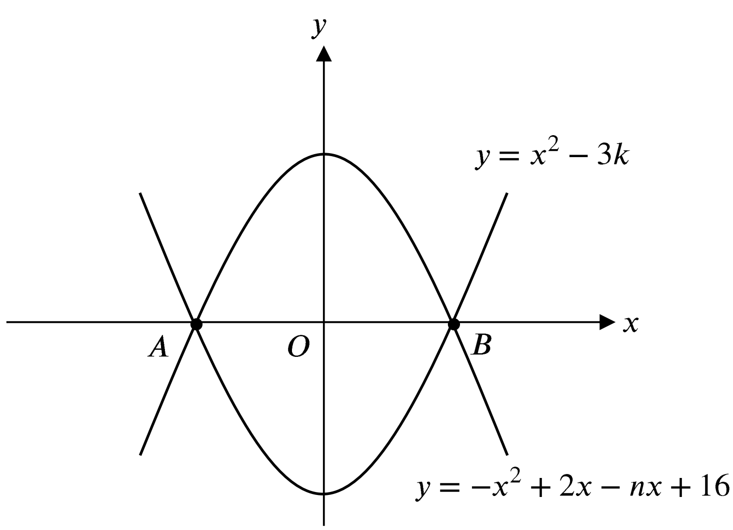
[4 *marks*]

2 It is given that the quadratic equation $x^2 + px - \frac{pq}{2} = qx$ has real roots.

Show that the statement above is true for all values of p and q. Prove your answer mathematically.

[5 *marks*]

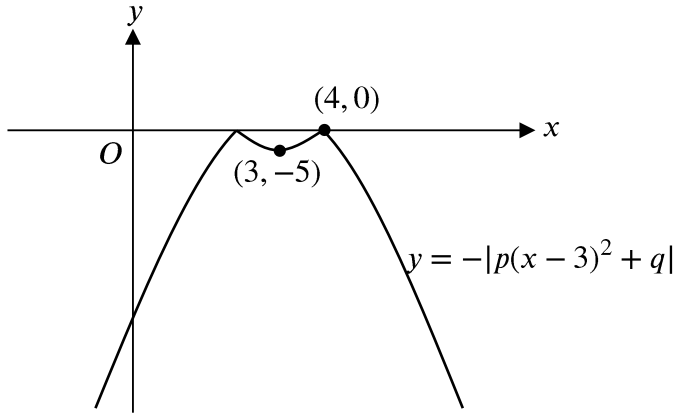
3 Diagram below shows two graphs of the curves which are intersecting at the points A and B on the x-axis.



Given that both graphs have the same axis of symmetry. Find the values of n and k.

[4 *marks*]

4 Diagram below shows part of the curve $y = -|p(x-3)^2 + q|$, where $p < 0$. The point $(3, -5)$ is the turning point of the curve.



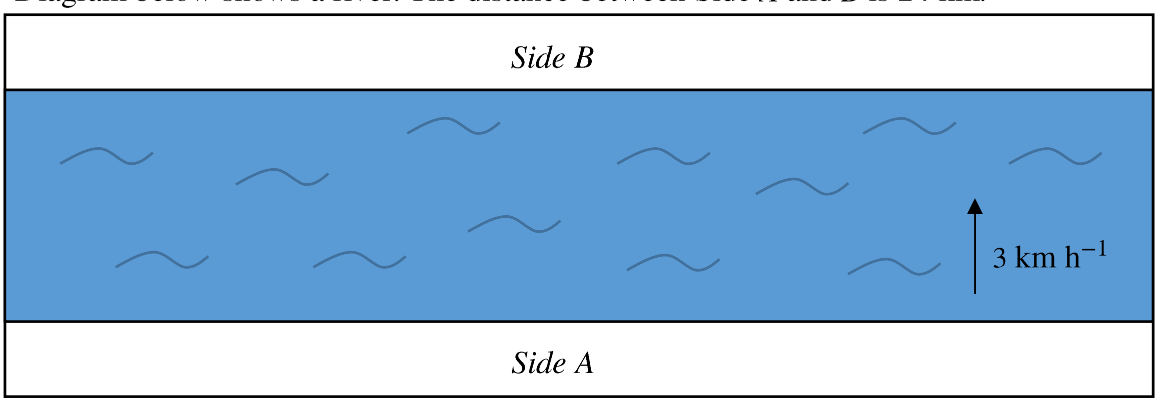
Find

(*a*) the value of p and q,

(*b*) the range of values of y for the domain $3 \leq x \leq 6$.

[5 *marks*]

5 Diagram below shows a river. The distance between Side A and B is 24 km.

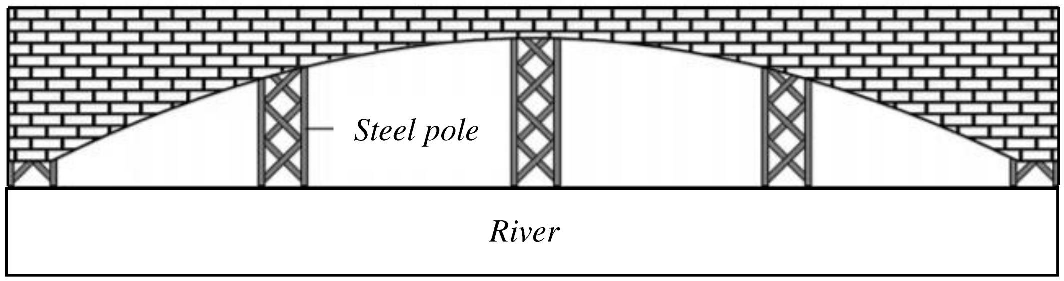


Amir is on Side A. He wants to get across the river and reach Side B by riding a boat. When he rides the boat to get to Side B, there is a water current with a velocity of 3 km h^{-1} flowing in the direction of the boat is heading. After staying at Side B for some time, he then uses the boat to return to Side A. The to-and-fro journey took 6 hours.

Given the boat maintains its uniform velocity and the water current always flow in one direction only throughout the whole journey. Find the velocity, in km h^{-1}, of the boat.

[5 *marks*]

6 Diagram below shows the side view of a bridge supported by five steel poles that are built in a symmetrical manner. The height, in m, of the curvature of the bridge from the river level is represented by the function $h(x) = ax^2 + \frac{1}{2}x + c$ where x is the distance, in m, of a pole from the first pole, while both a and c are constants.



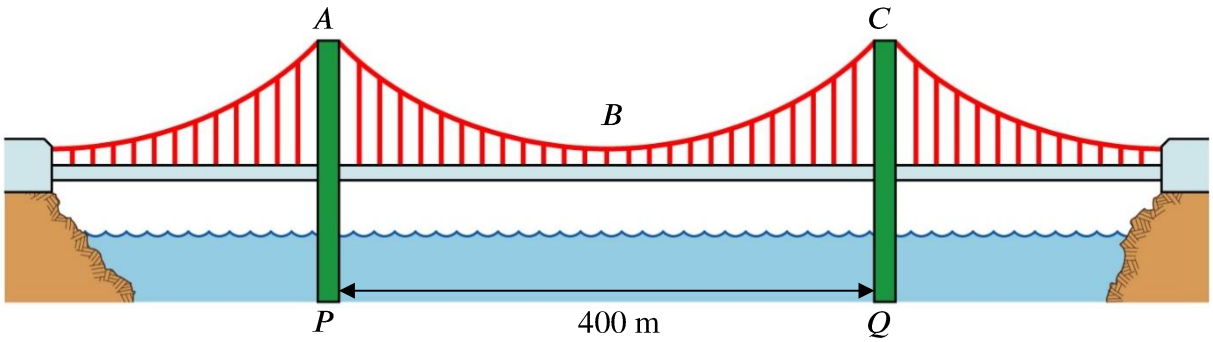
Given the length of the bridge is 200 m and the height of the first pole is 5 m.

(*a*) Find the height, in m, of the highest steel pole.

(*b*) During the Monsoon season, the second highest steel poles are completely submerged by the river when the river level rose by 20 m.
Find the distance, in m, between the first pole and the second highest steel poles.

[7 *marks*]

7 The diagram below shows a suspension bridge of length 400 m across river bed PQ. The height, h, in m, of the supporting cable ABC from the river bed can be represented by the equation $h(x) = \frac{1}{200}x^2 + bx + c$, where x is the distance from P, in m, while b and c are constants.



(a) Find the value of b.

(b) For safety precautions, the lowest point of the supporting cable needs to be at least 75 m above the river bed. Find the minimum value of c such that the requirement is met.

(c) During winter, the supporting cable of the bridge contracted due to the cold weather. This causes the supporting cable ABC to straighten and now passes through the point (100, 145). Given that the two end of the supporting cable ABC remains at the same location after the contraction, find the equation that represents the contracted supporting cable ABC.

[8 *marks*]

SYSTEM OF EQUATIONS

1 Solve the following system of linear equations. Hence, deduce the results of the findings.

$$\begin{aligned} y - 2z &= 4 \\ 2x - 3y + 2z &= -2 \\ -4x + 5y - 2z &= 6 \end{aligned}$$

[4 *marks*]

2 Given the following systems of equations has no solutions.

$$\begin{aligned} 2px + 2y - 6z &= 2 \\ qx + 2y - 8z &= 7 \\ 3x - 2z &= 12 \end{aligned}$$

Express q in terms of p.

[7 *marks*]

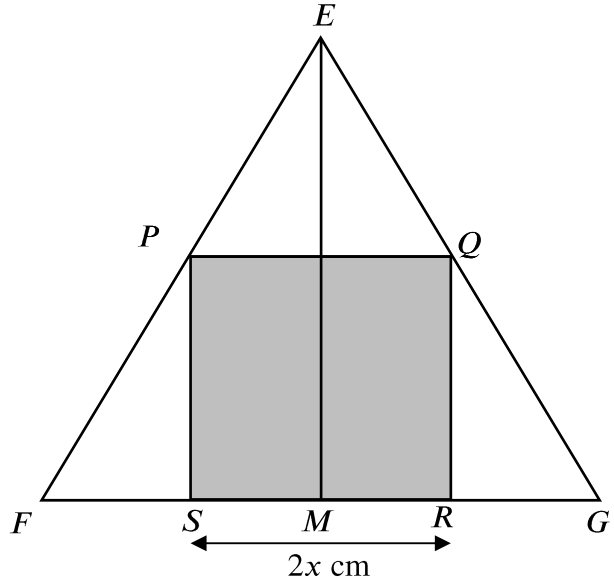
3 Fifi, a chemist, have three types of solutions, solution ***A***, ***B*** and ***C***. Solutions ***A***, ***B*** and ***C*** are solutions with an acid concentration of 10%, 40% and 60% respectively.

One day, she prepared a 10 litres standard solution with an acid conentration of 45% from the three solutions. It is given that amount of solution A she used is twice the amount of solution ***B*** used.

Calculate, in litres, of the amount of solution ***A***, ***B*** and ***C*** that is used to prepare the standard solution.

[8 *marks*]

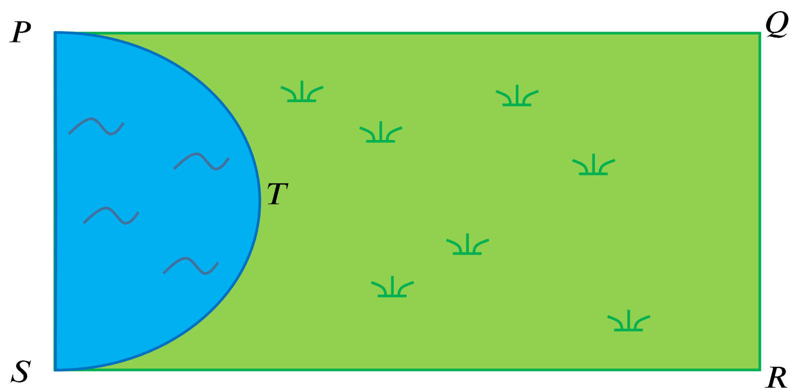
4 The diagram below shows a triangle EFG with a rectangle $PQRS$ inside it. It is given that $EF = EG = 17$ cm, $QR = y$ cm and $FG = 16$ cm. M is the midpoint of FG.



If the area of rectangle $PQRS$ is 45 cm^2, find the values of x and y.

[7 *marks*]

5 Diagram below shows the plan of a rectangular garden $PQRS$. The garden consists of a semicircular pond PTS and grassy area $PQRST$.

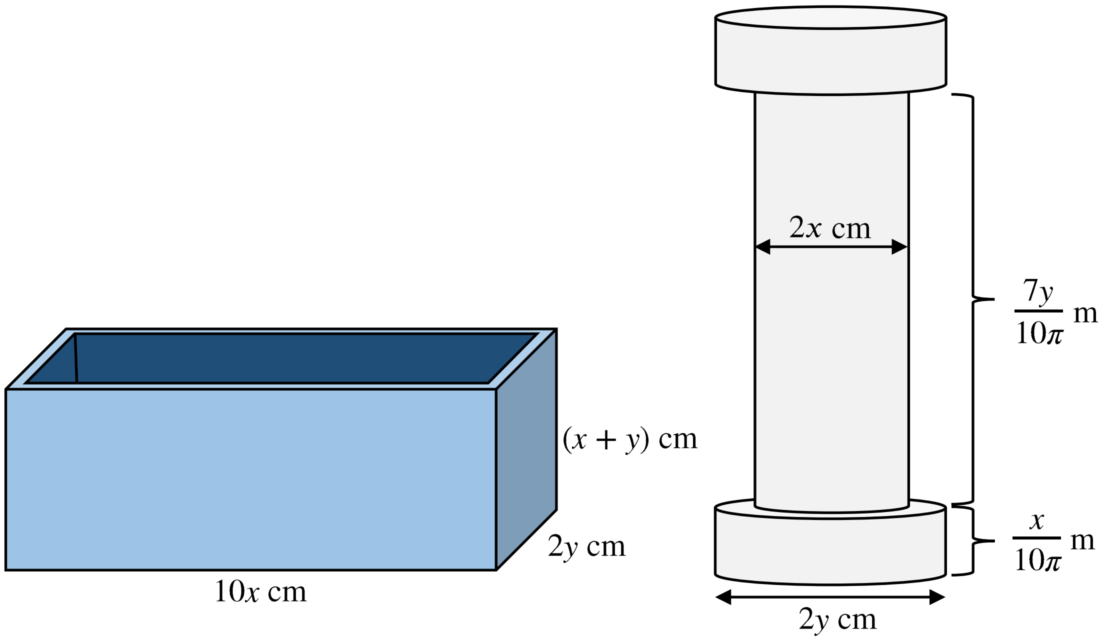


It is given that $SR = 6y$ m and $QR = 7x$ m, $x \neq y$. The area of the rectangular garden $PQRS$ is 168 m^2 and the perimeter of the grassy area is 60 m. The pond with a uniform depth contains 15.4 m^3 of water.

By using $\pi = \dfrac{22}{7}$, find the depth, in m, of water in the pond.

[8 *marks*]

6 Diagram below shows a cuboid container and a concrete pillar.



The Merbau District Council has decided to build a community hall. During the construction, the concrete pillars are built to prevent the community hall from collapsing. The pillars are made up of a cylinder with a diameter of $2x$ cm and a height of $\frac{7y}{10\pi}$ m, along with two other identical cylinders with a diameter of $2y$ cm and a height of $\frac{x}{10\pi}$ m as shown in the diagram above.
It is given that two cuboid container in the diagram above, each with a perimeter of 1240 cm, needs to be completely filled with concrete mixture to build one pillar.

Find the value of x and y that satisfies the situations above.

[8 *marks*]

INDICES, SURDS AND LOGARITHMS

1 Determine the number of digits of a if $a = 2^{165} \times 5^{168}$.

[4 *marks*]

2 If $2^p = 3^q = 36^r$.

Show that $2r(p + q) = pq$.

[3 *marks*]

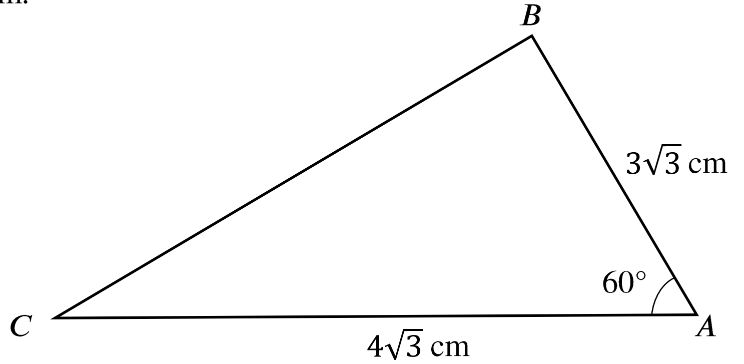
3 Given $\left(x^2 - 5x + 5\right)^{(x^2 - 11x + 30)} = 1$. Find all possible values of x.

[6 *marks*]

4 Solve the equation $\dfrac{\frac{3}{\sqrt{x}} - 5x^{1.5}}{\sqrt{x} - \frac{3}{\sqrt{x}}} - x = 0$

[4 *marks*]

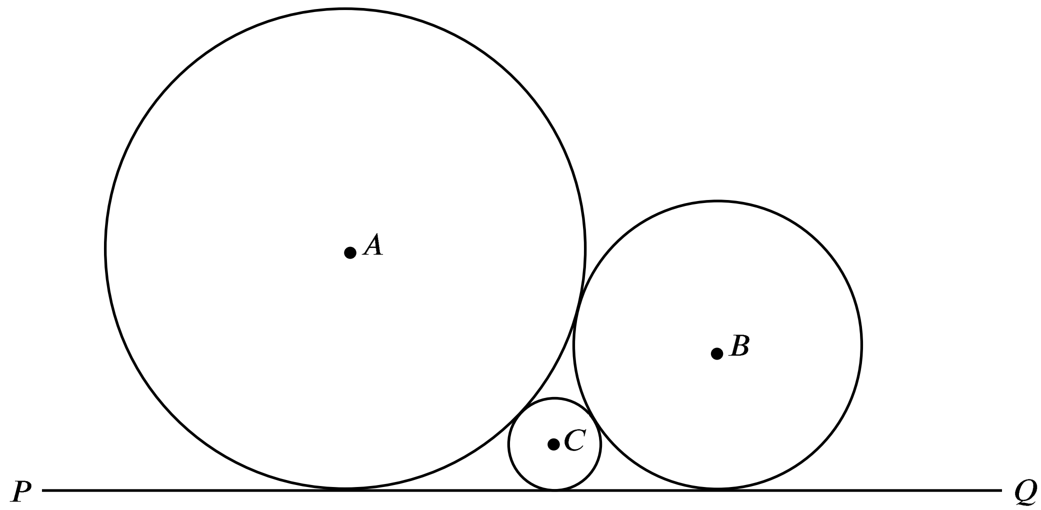
5 Diagram below shows triangle ABC where $\angle BAC = 60°$, $AB = 3\sqrt{3}$ cm and $AC = 4\sqrt{3}$ cm.



Without finding the value of $\cos \angle BAC$, find the length of BC.

[5 *marks*]

6 Diagram below shows three circles with centre A, B and C respectively.



It is given that line PQ is tangent to all three circles and the circles are tangent to each other. The diameter of the circle with centre A and centre B is 60 cm and 36 cm respectively.

Calculate the radius of the circle with centre C, in cm, in form of $a + b\sqrt{c}$, where a, b and c are integers.

[8 *marks*]

7 Find the value of $\log_2\left(\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}}\right)$.

[4 *marks*]

8 It is given that $\log_x y = \log_y x$ and $\log_x(x - y) = \log_y(x + y)$, where $x > 0,\ x \neq 1$ and $y > 0,\ y \neq 1$.

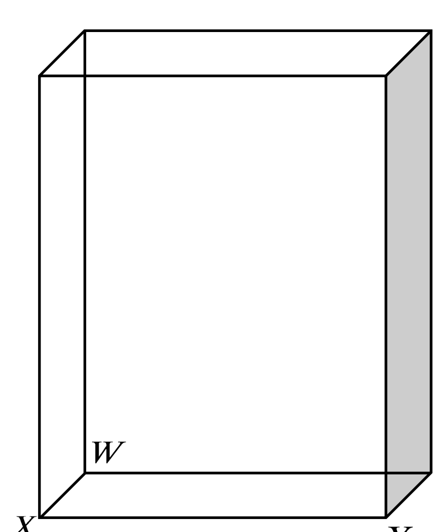
Show that $x^4 - x^2 - 1 = 0$.

[5 *marks*]

9 Given that $\log_m p^2q = x$ and $\log_m \frac{q}{p} = y$, express $\log_m pq$ in terms of x and/or y.

[4 *marks*]

10 Diagram below shows a cuboid with a base $WXYZ$, the length of HZ and XY is given by $(4\sqrt{6} + 3\sqrt{2})$ cm and $(2 + \sqrt{6})$ cm respectively.



Given that the length, in cm, of YZ is $\left(\frac{2}{1 + \log_a bc} + \frac{2}{1 + \log_b ac} + \frac{2}{1 + \log_c ab}\right)$.

Where a, b and c are constants.

(*a*) Show that $YZ = 2$ cm.

(*b*) Find the length, in cm, of XZ, in form of $p\sqrt{q} + \sqrt{r}$, where p, q and r are integers.

(*c*) Hence, given the value of $(HX)^2$, in cm^2, is $2^{x+y} + (2^{2x-y})\sqrt{6} + 48\sqrt{3}$. Determine the value of x and y.

[10 *marks*]

PROGRESSIONS

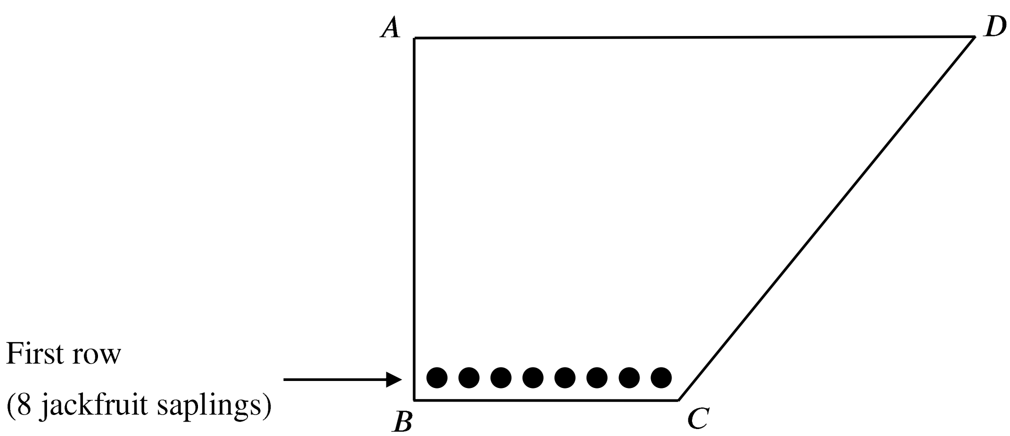
1 Given $\sin\theta$, $2\cos\theta$ and $2\sin\theta$ are three consecutive terms of an arithmetic progression, and $0° < \theta < 90°$. Calculate the value of θ.

[3 *marks*]

2 If $\log_{10} x + \log_{10} x^2 + \log_{10} x^3 + \log_{10} x^4 + \cdots + \log_{10} x^n = n(n+1)$. Calculate the value of x.

[4 *marks*]

3 Diagram below shows a cultivation plan of jackfruit saplings on a trapezium-shaped plot of land $ABCD$ belonged to Mr. Shulaimi. According to the plan, the first row can be planted with 8 saplings, the next line increases by 4 saplings, every jackfruit sapling occupies an area with a dimension of 3 m × 3 m.



After planting all of the jackfruit saplings, the saplings occupied an area of 1260 m^2. Determine the length CD, in m.

[6 *marks*]

4 Diagram below shows the position of A and B on a straight line with distance $AB = 920$ m.

Particle P moves from point A towards point B with an initial velocity of 10 m s^{-1} and its velocity increases by 2 m s^{-1} for each of the following second. Whereas particle Q moves at a constant velocity of 8 m s^{-1} from point B to point A. Given that both particles move at the same time.

Find

(a) the value of t, if particles P and Q meet after t seconds.

(b) the distance between P and Q by the time particle Q reaches point A.

[7 *marks*]

5 A sequence of integers can be formed as:

$$\{(1), (2, 3), (4, 5, 6, 7), (8, 9, 10, \dots, 15), (16, 17, 18, \dots, 31), \dots, (a, a+1, a+2, \dots, l)\}$$

Given $(a, a+1, a+2, \dots, l)$ is the set of integers in the n^{th} bracket.

(a) Express in terms of n,

i) the number of integers in the n^{th} bracket.

ii) a.

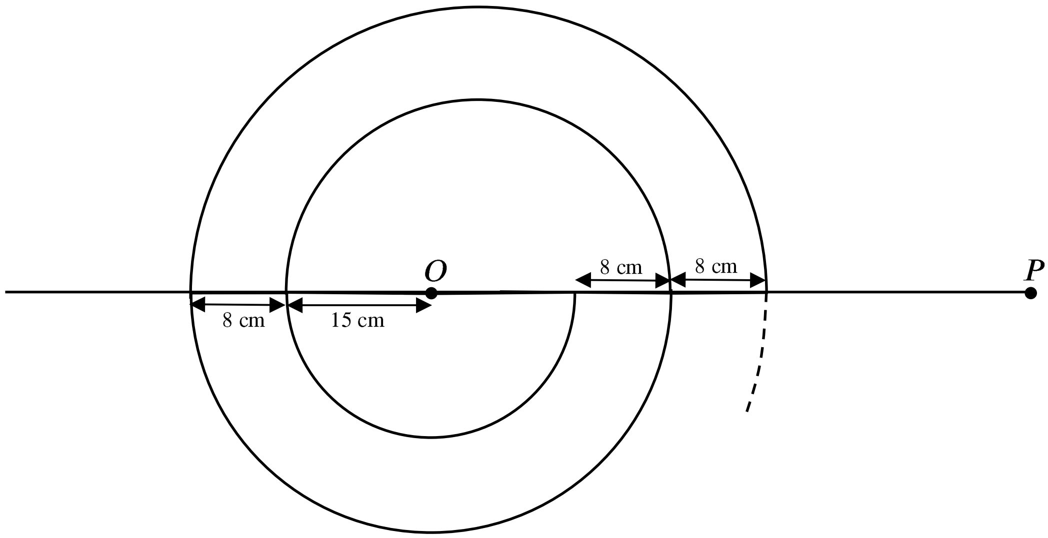
iii) l.

Show your calculations.

(b) Hence, show that the sum of integers in the n^{th} bracket is $2^{n-2}[3(2^{n-1}) - 1]$.

[7 *marks*]

6 Diagram below shows a spiral pattern obtained when a piece of wire is bent into several semicircles, with the condition such that the distance between consecutive points on the circumference must be 8 cm as shown in the diagram below.



It is given that the radius of the first semicircle, with centre O, has a radius of 15 cm.

(a) Determine which semicircle has an arc length of 131π cm.

(b) If $OP = 215$ cm. Find the length of wire needed, in terms of π, such that the end of the wire touches point P.

[8 *marks*]

7 Given T_n is the n^{th} term of an arithmetic progression, and $U_n = 2^{3(4 - T_n)}$.

(a) Show that U_n is the n^{th} term of a geometric progression.

(b) Hence, find the common ratio of the geometric progression if $T_n - T_{n-1} = 2$.

[6 *marks*]

8 Show that $\frac{1}{5} + \frac{2}{25} + \frac{3}{125} + \frac{4}{625} + \cdots = \frac{5}{16}$.

[4 *marks*]

9 The first seven terms of a geometric progression are $T_1, T_2, T_3, T_4, T_5, T_6, T_7, \ldots$. Given the sum to infinity of the odd terms is $\frac{27}{4}$ and the sum to infinity of the even terms is $\frac{7}{4}$.

Find the first term and the common ratio of the progression.

[4 *marks*]

10 An insurance company offers a saving plan where customers only have to deposit their savings for 10 years. The deposit saving for 10 years form an arithmetic progression. Given that the monthly deposit for the first three years are RM 200, RM 250 and RM 300. Other information about this saving plan are given as in the diagram below.

Kah Meng and Jessica began saving their money under this saving plan from 1st January 2000.

(*a*) Kah Meng wishes to stop saving and withdraw all of his money on 31st December 2004. How much money will he get back from the insurance company?

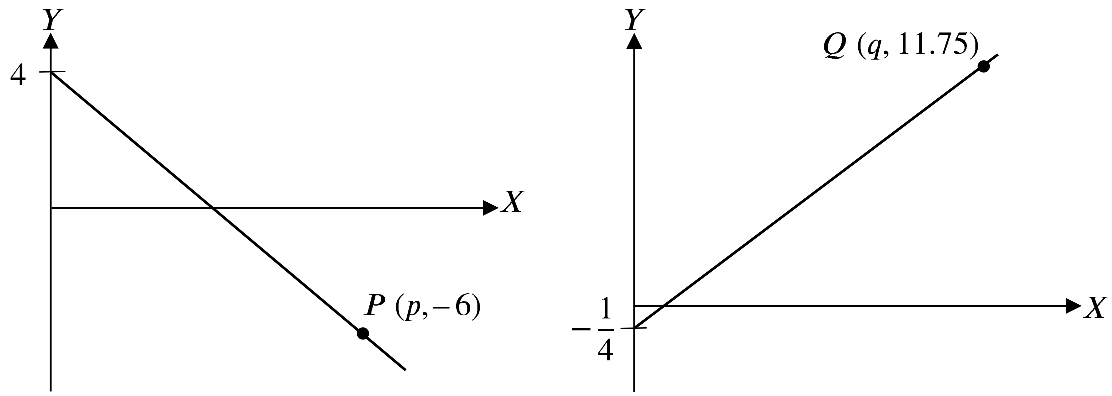
(*b*) Jessica wishes to save money under this plan until 31st December 2017, and withdraw all of her money then.

How much money in total that Jessica will get?
What is the average annual interest rate that Jessica gets from this saving?

[7 *marks*]

LINEAR LAW

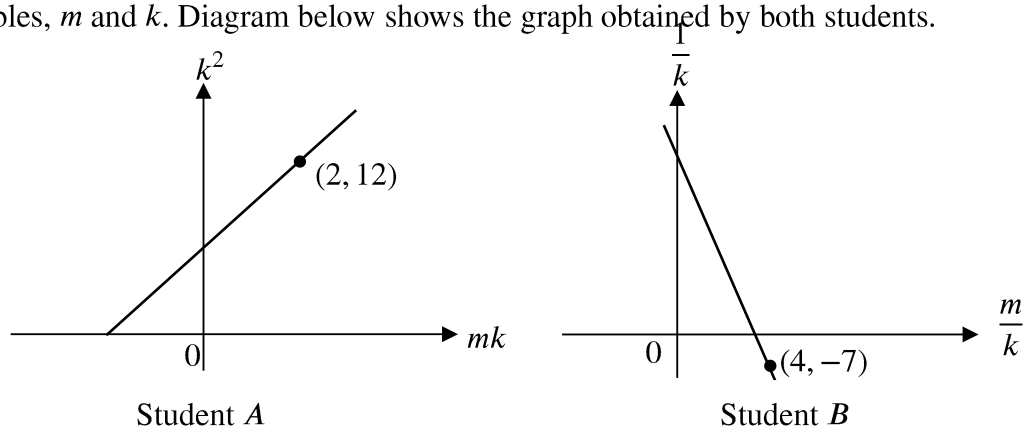
1 The variables x and y are related by the equation $y = \frac{4}{\sqrt{x}} - \frac{\sqrt{x}}{4}$, where p and q are constants. Diagram below shows the different graphs obtained when the equation is expressed in its linear form.



Find the value of x at point P and Q respectively.

[5 *marks*]

2 An experiment is carried out by two students to determine the relationship between two variables, m and k. Diagram below shows the graph obtained by both students.

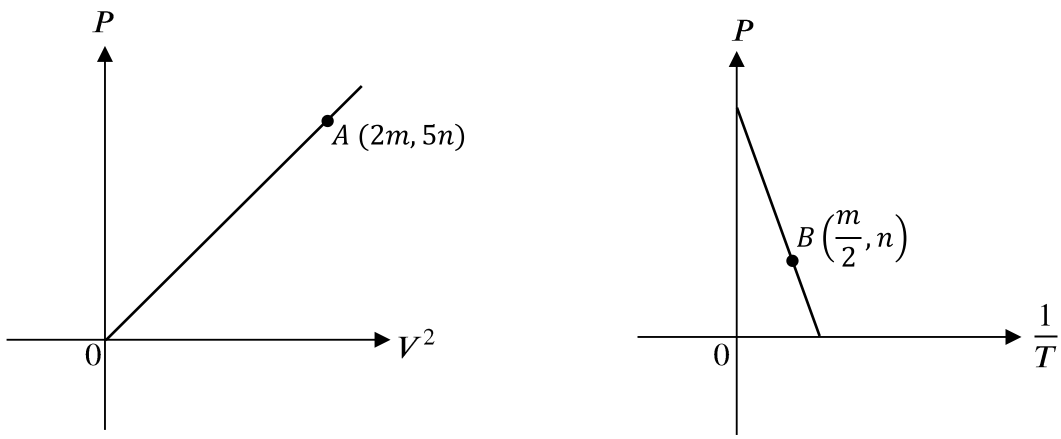


The students used linear law to express the same equation into forms that are suitable to draw the straight lines above.

Express m in terms of k.

[5 *marks*]

3 Diagram below shows the relationships between three variables P, V and T in form of graph.



The relation between variables P, V and T is represented by the equations $\dfrac{P}{V} = 5V$ and $\dfrac{1}{P} = \dfrac{T}{9T - 5}$.

(*a*) Determine the value of m and n.

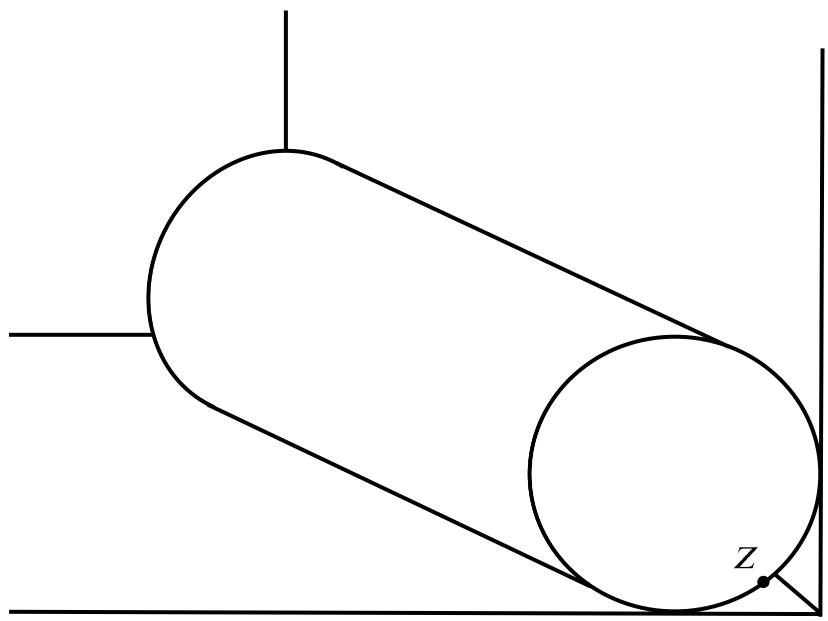
(*b*) Hence, sketch the graph of V^2T against T. Your graph should include the V^2T-intercept and the corresponding coordinates of point A and B.

[8 *marks*]

COORDINATE GEOMETRY

In this section, solution by scale drawing is **not** accepted.

1 Diagram below shows a cylindrical container with a length of 20 cm place on the floor against the wall. Point Z is the point on the edge of the base of the container. It is given that the distance of point Z is 2 cm from the wall and 1 cm from the floor.



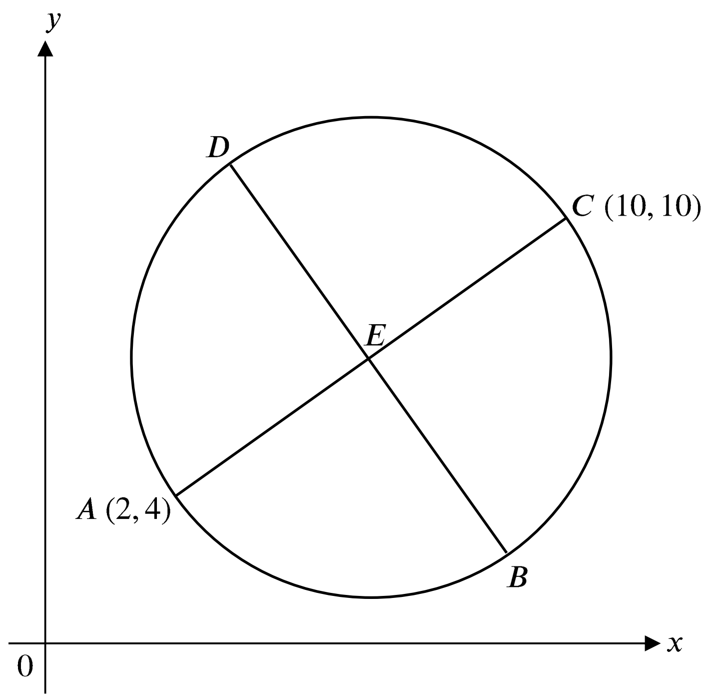
Mira wants to put the container inside a shoe box with dimension 21 cm $\times$ 7 cm $\times$ 7 cm. Determine whether the container can fit inside the shoe box.

[6 *marks*]

2 It is given that point $A\ (p, q)$, $B\ (3, \frac{q}{2})$ and $C\ (3p, 8)$ lies on the Cartesian Plane. Express p in terms of q if triangle ABC cannot be formed.

[3 *marks*]

3 In the diagram below, AC and BD are the diameter of the circle with centre E. The coordinates of points A and C are $(2, 4)$ and $(10, 10)$ respectively. Given $AD = DC$.



(a) Determine the coordinate of point D.

(b) A quadrilateral is formed from joining the points A, B, C and D. Show that the quadrilateral formed is a square, show your working to prove your answer.

[10 *marks*]

4 $P\ (h, 8)$ and $Q\ (k, 2)$ are two points on the curve $y = \dfrac{8}{x}$.

(a) Find the equation of line PQ.

(b) Using coordinate geometry methods, find the equations of tangent to the curve that are parallel to PQ.

[7 *marks*]

5 Point P moves such that its distance from $(2, 0)$ and the line $x = -3$ is the same. Determine the equation of locus P.

[3 *marks*]

6 The coordinates of the vertices of ΔJKL are $J(2,-1), K(11,5)$ and $L(5,9)$. Point $P\ (a,b)$ is located within ΔJKL such that the areas of ΔJKP, ΔKLP, and ΔJLP are equal.

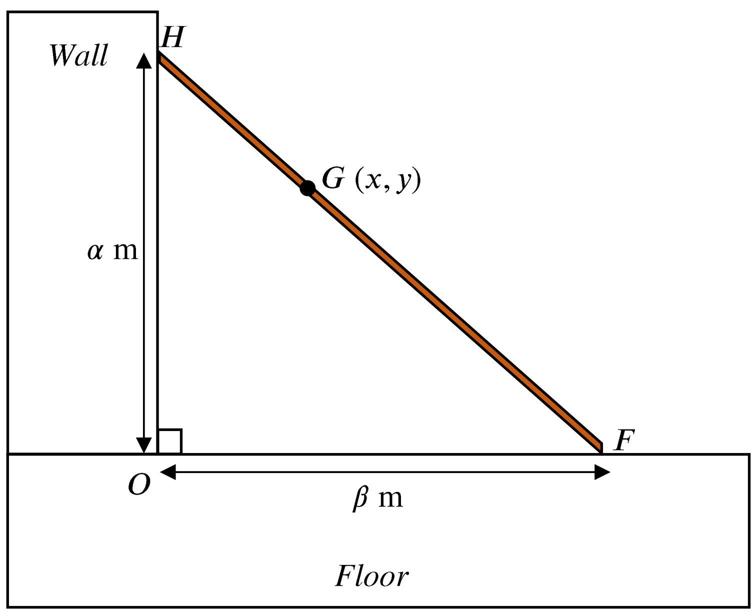
Find

(a) the area of ΔJKL,

(b) the value of a and b.

[6 *marks*]

7 Diagram below a pole, FH of length 1.5 m is leaning against a wall touches the floor and wall at points H and F respectively. Point $G\ (x, y)$ is a point on the pole such that the ratio $HG : GF = 1 : 2$. Point O is the origin with coordinate $(0, 0)$.



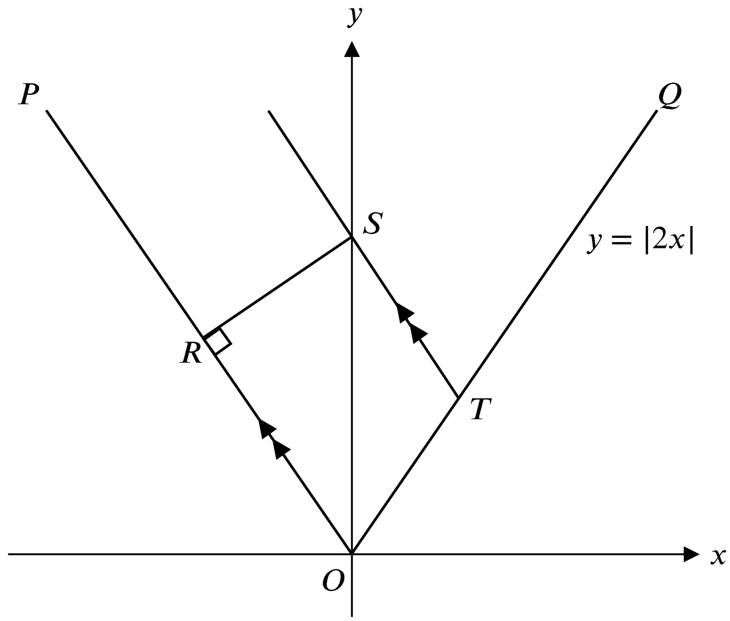
Given 1 m represents 1 unit. If the pole begins sliding down, show that the locus of the moving point G follows the equation $4x^2 + y^2 = 1$.

[4 *marks*]

8 Calculate the area, in units^2, of the area enclosed by the graph with the equation $6|x| + 2|y| = 18$.

[4 *marks*]

9 In the diagram below, POQ is the graph of $y = |2x|$. R is a point on OP and O is the origin. RS is perpendicular to OP and OR is parallel to TS. Given the length of OR and TS are $8\sqrt{5}$ units and $5\sqrt{5}$ units respectively.



(a) Show that Point S lies on the y-axis.

(b) Hence, calculate the area, in units^2, of trapezium $ORST$.

[10 *marks*]

10 Circle J has a centre at point $W(5, 3)$. It is given that circle J intersects the y-axis at one point and x-axis at two points.

Calculate the area, in units^2, of the area of triangle formed by these three intercepts.

[4 *marks*]

11 Given that line PQ has an equation $5y - 3x = 45$. Line RS, which has an integer for its y-intercept, is perpendicular to line PQ, such that it intersects line PQ at the second quadrant.

Prove that, only a total of 33 possible line RS exists.
(Intersection at one of the axes do not count.)

[5 *marks*]

12 A circle with centre A has the equation $(x - 8)^2 + (y - 4)^2 = 100$

(a) Show that point $T(-6, 6)$ is outside the circle.

(b) Two tangents from $T(-6, 6)$ are drawn to the circle with centre A.
Show that the angle between one of the tangents and CT is 45°.

(c) The two tangents from $T(-6, 6)$ touches the circle with centre A at point F and G.

i) Find the equation of line FG, in form of $y = mx + c$.
ii) Hence, determine the x-coordinate of F and G.

[10 *marks*]

13 Three lines are drawn between points A $(1, 2)$, $B(3, 6)$ and $C(4, -1)$ to form a triangle. Perpendicular bisector of AB, BC and CA are constructed and intersects at point P. A circle with centre P is drawn with radius AP.

(a) Prove that the vertices of the triangle lies on the circle. Show your calculations.

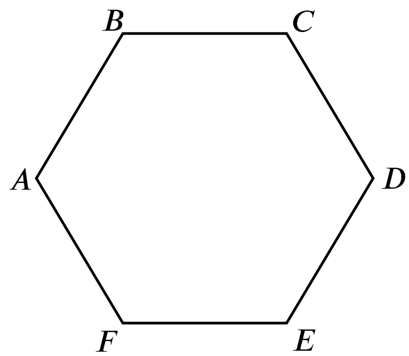
(b) Hence, show that the property of the circle in (a) is true for all coordinates of A, B and C as long as triangle ABC can be formed.

[6 *marks*]

VECTORS

In this section, solution by scale drawing is **not** accepted.

1 Diagram below shows a regular hexagon $ABCDEF$.



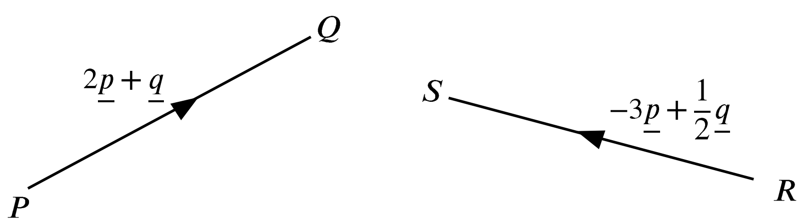
Given $\overrightarrow{FE} = \underset{\sim}{a}$ and $\overrightarrow{ED} = \underset{\sim}{b}$. Express the following in terms of $\underset{\sim}{a}$ and/or $\underset{\sim}{b}$.

(*a*) $\overrightarrow{AB} + \overrightarrow{BD}$

(*b*) $\overrightarrow{AD} + \overrightarrow{FA} - \overrightarrow{CD}$

[4 *marks*]

2 Diagram below shows two vectors, $\overrightarrow{PQ}$ and $\overrightarrow{RS}$, expressed in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$.

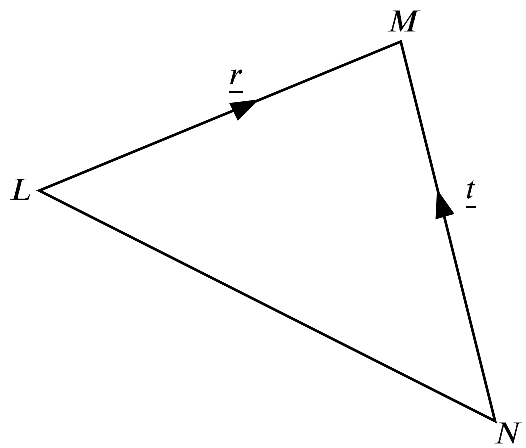


(*a*) Show that $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are two vectors that are not parallel to each other.

(*b*) Hence, given $(2m - 4)\overrightarrow{PQ} = (11 + k)\overrightarrow{RS}$. Determine the value of m and k.

[4 *marks*]

3 Diagram below shows an isosceles triangle LMN. Given $|\underset{\sim}{r}| = |\underset{\sim}{t}| = 13$ units, and $|\overrightarrow{LN}| = 18$ units.



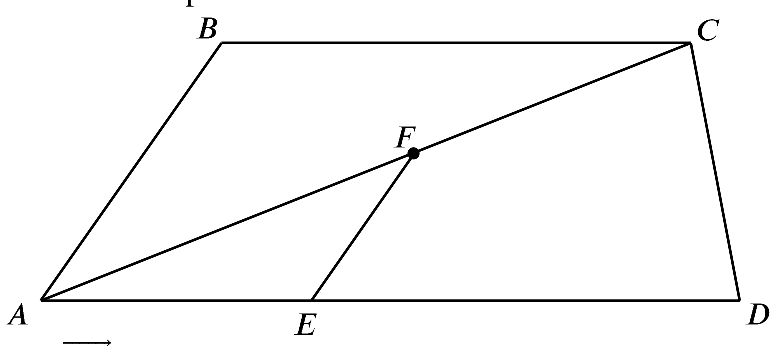
Without finding any of the angles in triangle LMN, find $|\underset{\sim}{r} + \underset{\sim}{t}|$.

[3 *marks*]

4 Given vector $\underset{\sim}{p} = (h-1)\underset{\sim}{i} + 2\underset{\sim}{j}$ is perpendicular to vector $\underset{\sim}{q} = 8\underset{\sim}{i} + k\underset{\sim}{j}$.
Express h in terms of k.

[3 *marks*]

5 Diagram below shows trapezium $ABCD$.

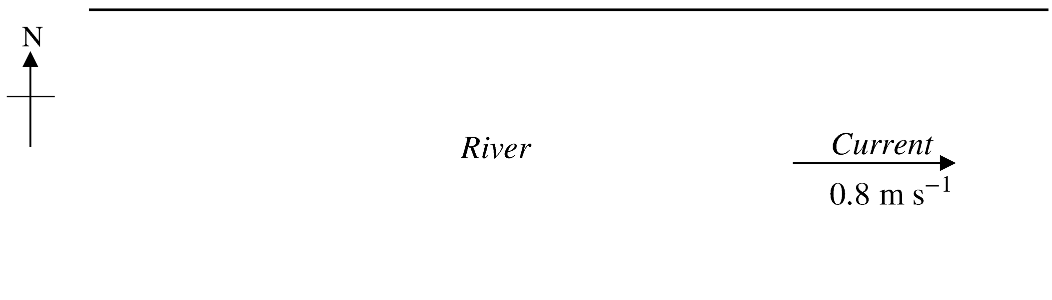


Given $\overrightarrow{AB} = \underset{\sim}{x}$, $\overrightarrow{AD} = \underset{\sim}{y}$ and $3AD = 5BC$. Point E lies on line AD such that $AE : ED = m : n$. Point F is the midpoint of AC and AB is parallel to EF.

Express m in terms of n.

[5 *marks*]

6 Diagram below shows a sketch of a river.



A swimmer is swimming across the river towards the north. There is a current with a constant velocity of 0.8 m s^{-1} flowing towards the east as shown in the diagram.

(*a*) If the swimmer swims with a velocity of 1.8 m s^{-1}. Calculate the resultant speed of the swimmer.

(*b*) Given the river is 20 metres wide, calculate the velocity of the swimmer and hence state its direction so that he eventually reaches directly opposite the river bank in 10 seconds.

[4 *marks*]

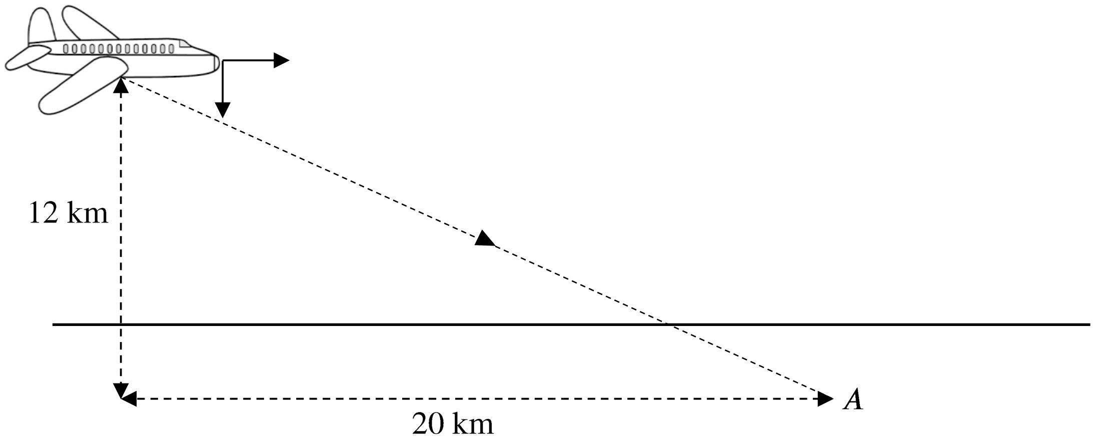
7 Aman and his friend Ben leave from town ***A*** and town ***B*** at the same time with a velocity $v_a = 12\underline{i} + 8\underline{j}$ km h^{-1} and $v_b = -6\underline{i} + 6\underline{j}$ km h^{-1} respectively. Given the position vector of town ***B*** from town ***A*** is $45\underline{i} + 5\underline{j}$ km.

(*a*) After t hours, state the position vector of Ben from Amir, in terms of t, $\underline{i}$ and $\underline{j}$.

(*b*) Hence, determine whether Aman and Ben will meet each other.
Show your calculations.

[5 *marks*]

8 Diagram below shows a plane flying in the air.

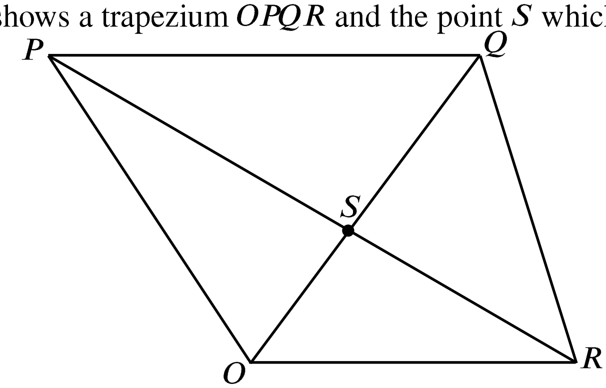


The plane is at 12 km above the ground when it starts to land on an airport at A which is 20 km away. At this instant, the aircraft is cruising at a constant velocity of 90 km h^{-1} horizontally and descending vertically at a constant velocity of r km h^{-1}.

If the plane eventually lands on the airport in 10 minutes, calculate the value of r.

[4 *marks*]

9 Diagram below shows a trapezium $OPQR$ and the point S which lies on the line PR.



Given that $\overrightarrow{OP} = \underset{\sim}{a}, \overrightarrow{OR} = 2\underset{\sim}{b}$ and $3\overrightarrow{OR} = 2\overrightarrow{PQ}$.

(*a*) Express $\overrightarrow{PR}$ and $\overrightarrow{OQ}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

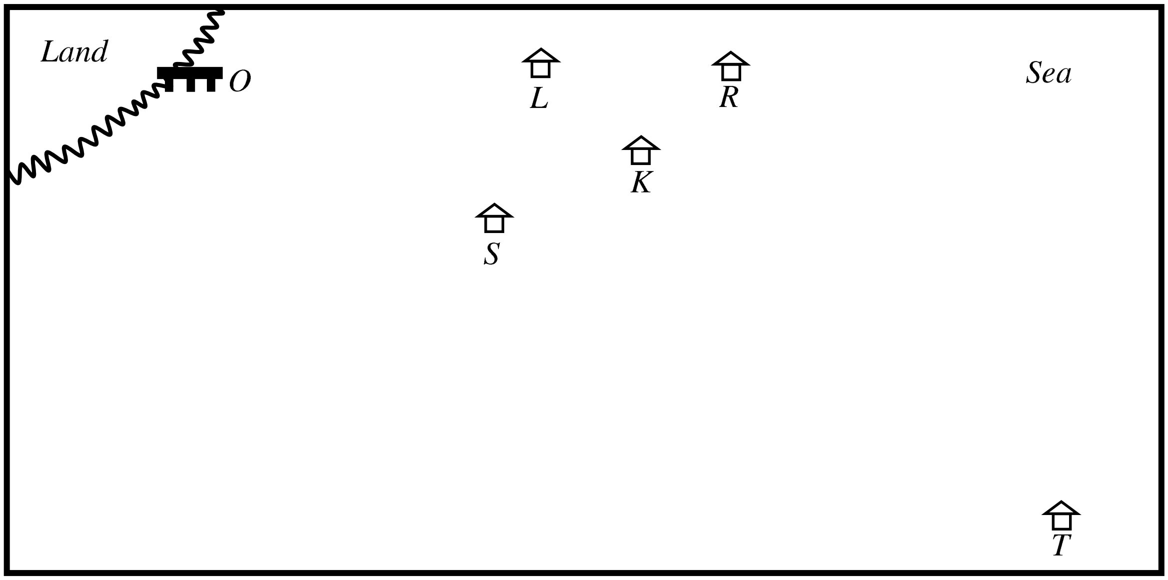
(*b*) Given $\overrightarrow{RS} = k\overrightarrow{RP}$, where k is a constant.

i) Find the value of k if the points O, S and Q are collinear.

ii) Hence, given the area $\Delta ORS = 15$ cm^2, find the area ΔOSP.

[8 *marks*]

10 Diagram below shows the position of jetty O and kelongs K, L, R, S and T in the sea.



Kelong L is situated 400 m from jetty O and kelong R is situated 600 m from jetty O in the direction of OL. Kelong S is situated 300 m from jetty O and kelong T is situated 600 m from kelong S in the direction of OS. Kelongs L, K and T are situated on a straight line such that the distance of kelong K from kelong T is 5 times its distance from kelong L.

By using $\underset{\sim}{p}$ to represent 100 m in the direction of OR and $\underset{\sim}{q}$ to represent 150 m in the direction of OT.

(*a*) Express in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$,

i) $\overrightarrow{OK}$

ii) $\overrightarrow{RK}$

(*b*) If Joe uses a binocular to observe kelong R from kelong S, determine whether kelong R can be seen without being blocked by kelong K or otherwise.

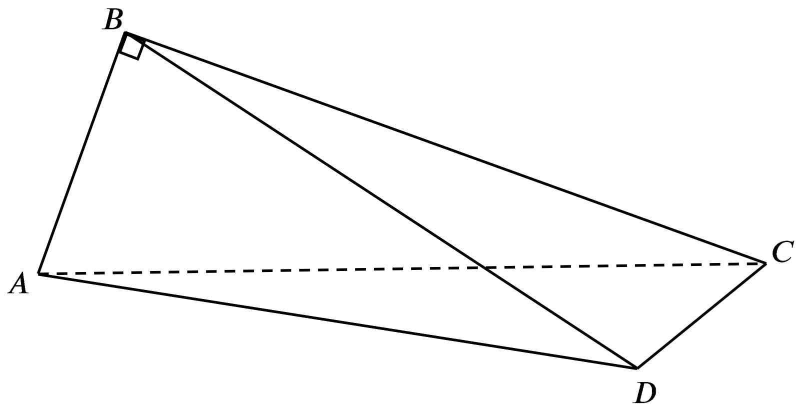
Prove your answer mathematically.

[8 *marks*]

SOLUTION OF TRIANGLES

In this section, solution by scale drawing is **not** accepted.

1 Diagram below shows a tetrahedron $ABCD$ such that $\angle BAC = 64°, \angle ACD = 35°$, $\angle BDC = 104°$, $AB = 8$ cm and $BD = 15$ cm.



It is given that the area of triangle BCD is 29 cm^2 and ABC is a right-angled triangle.

(*a*) Calculate

i) the length, in cm, of CD,

ii) the length, in cm, of AD,

iii) $\angle CAD$.

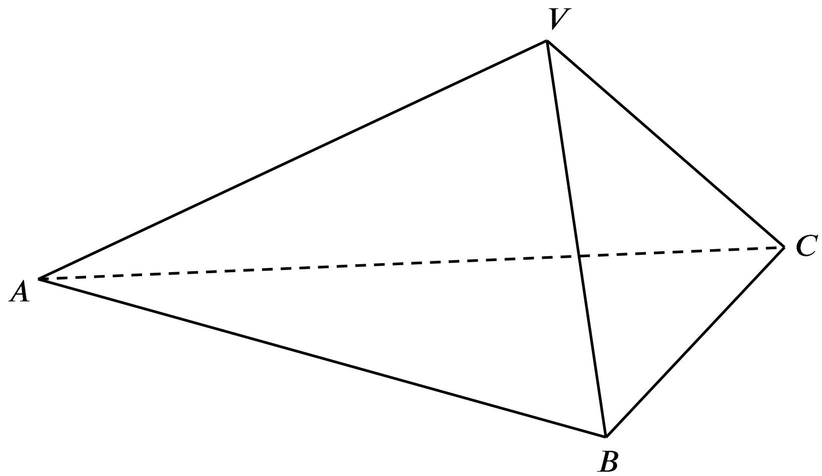
(*b*) Point C' lies on line AC such that $DC' = DC$.

i) Sketch triangle ADC'.

ii) Find the length, in cm, for AC'.

[10 *marks*]

2 Diagram below shows a tent $VABC$ in the shape of a pyramid with triangle ABC as the horizontal base, V is the vertex of the tent and the angle between the inclined plane VBC and the base is 50°.



Given that $VB = VC = 2.2$ m, $AB = AC = 2.6$ m. The area of the base is 3 m^2 and the angle between the straight line AV and the base is 25°. Calculate

(*a*) the length of BC,

(*b*) the length of AV,

(*c*) the angle between two inclined planes VAB and VAC.

[10 *marks*]

3 Rashid drove a boat westwards. He noticed a lighthouse at a distance of 25 km away at the beating of 235°. He continues to travel westward until the distance between him and the lighthouse is 16 km.

(*a*) Calculate the distance travelled by the boat between Rashid's first position and second position.

(*b*) Rashid continued to drive the boat westward until his distance from the lighthouse is 16 km again.

i) Find the distance between Rashid's second position and third position.

ii) Hence, determine the bearing of the lighthouse from the boat when the boat is at its third position.

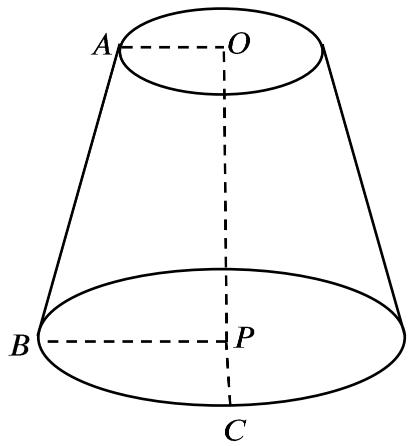
iii) At this position, Rashid wanted to go to the lighthouse. He thought about two of the routes that he could take.

Route A: Goes to the lighthouse directly from his current position. **Route B**: Backtracks until he reaches the point where the path from him to the lighthouse is the shortest. Then goes to the lighthouse.

Without calculating the actual distance covered by the two routes, determine which route is the better of the two.
Justify your answer.

[10 *marks*]

4 Diagram below shows a children's toy in the shape of a cone and its upper portion is cut off horizontally. The top and bottom surfaces with centre O and centre P are horizontal and the OP axis is vertical.



Given $OA = 5$ cm, $PB = 10$ cm, $OP = 16$ cm and $\angle BPC = 90°$.

Find

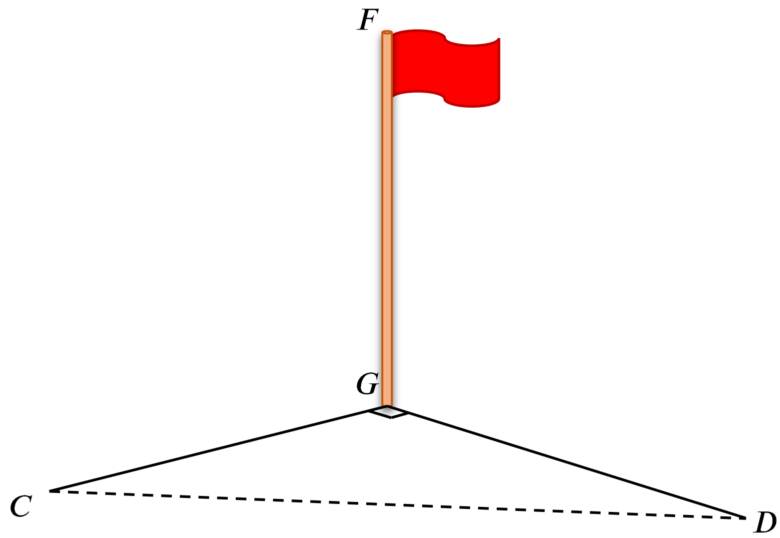
(a) the length of the straight line that joins points A and C,

(b) the area of plane ABC,

(c) the total surface area of the solid.

[10 *marks*]

5 Diagram below shows a vertical flag pole FG on a horizontal ground. Chee Seng and Daphine is located at point C and D respectively. There is a distance of 15 m in between them.



It is given that the angle of elevation of F from Chee Seng and Daphine are $44°$ and $31°$ respectively. They form a right-angle with the flag pole at $\angle CGD$.

(*a*) Calculate the height, in m, of the flag pole.

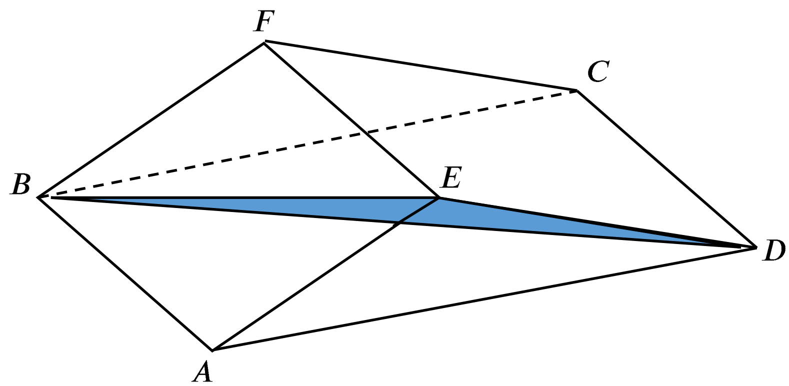
(*b*) Find the value of $\angle DCG$.

(*c*) The flag pole casts a shadow and the shadow intersects line CD such that the shadow length is the shortest distance between the flag pole and line CD.
Calculate the length, in m, of the shadow.

(*d*) Daphine moves away from the flag pole in the direction $\overrightarrow{CD}$ until point D' such that the angle of elevation of F from her decreases to $15°$.
Calculate the distance, in m, travelled by Daphine.

[10 *marks*]

6 Diagram below shows a transparent prism with a rectangular base $ABCD$. The inclined surface $ABFE$ is a square with sides 12 cm and the inclined surface $CDEF$ is a rectangle. AED is a uniform cross section of the prism. BDE is a blue coloured plane in the prism.



It is given that $\angle ADE = 37°$ and $\angle EAD = 45°$,

Find

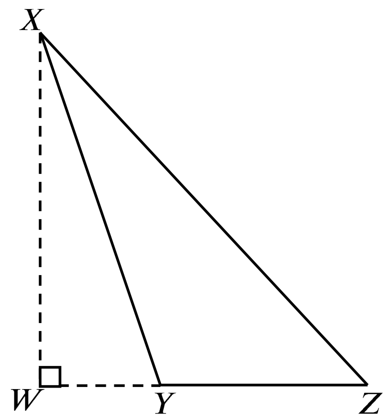
(*a*) the length, in cm, of DE,

(*b*) the area, in cm^2, of the blue coloured plane,

(*c*) the shortest length, in cm, from point E to the straight line BD.

[10 *marks*]

7 In the diagram below, WYZ is a straight line.



Given $XZ = 12$ cm, $YZ = 4$ cm, and $\sin \angle XYW = \frac{10}{11}$.

(a) Without using a calculator, find $\sin \angle YXZ$.

(b) Calculate the area of triangle XYZ. Hence, find the length of XW.

(c) Given $\angle XZY$ and the length of XW, XY and XZ is kept constant.

i) Determine the number of possible triangles exist for triangle XYZ and triangle WXZ if XW is the height of the triangle.
Explain your answer.

ii) P is a point such that $XP < XW$. Determine the number of triangle XZP that can be drawn if XW is the height of the triangle.
Justify your answer.

[10 *marks*]

INDEX NUMBERS

1 Table below shows the income and weightage in different economic sectors of country H.

Economic Sectors	Agriculture	Tourism	Manufacture	Quarrying
Income in year 2016 (H$)	210.3 Billion	582.5 Billion	r	k
Income in year 2020 (H$)	p	466.0 Billion	429.8 Billion	k
Income index in the year 2020 (2016 = 100)	130	q	140	s
Weightage	a	b	c	d

The ratio of the weightages of income from agriculture, tourism and manufacture is $x : y : z$.

(a) Calculate the value of p, q, r and s.

(b) Given the composite index for agriculture and tourism in year 2020 based on year 2016 is 120 whereas the composite index for agriculture and manufacture in year 2020 based on year 2016 is 125.

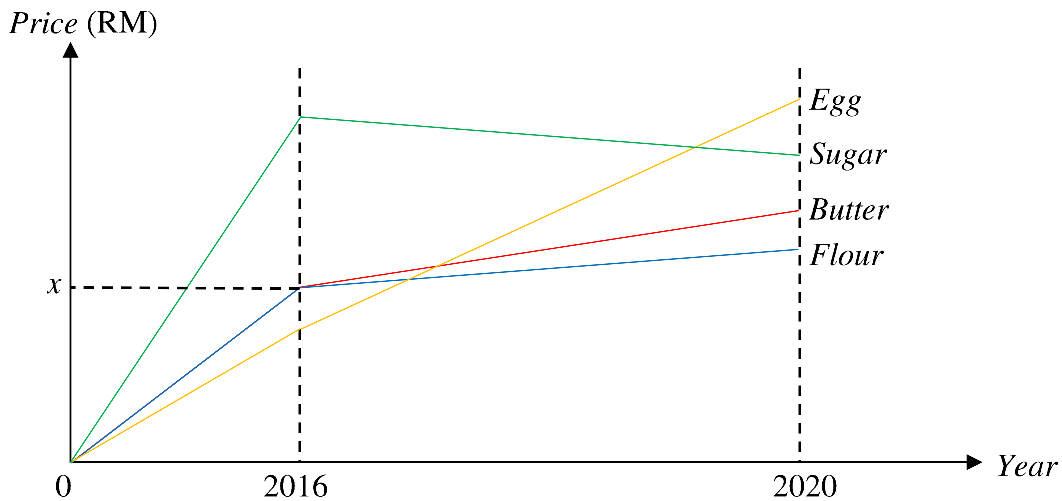
i) Determine the ratio $x : y : z$.

ii) Hence, if $x = a$, $y = b$ and $z = c$. Find the value of d if the composite index for the total income in year 2020 based on year 2016 is 107.

(c) The total income in year 2021 has increased by 25% compared to year 2016. Based on your answer in (b), calculate the percentage in increase or decrease of the total income in year 2021 compared to year 2020.

[10 *marks*]

2 Diagram below shows a graph and an incomplete table that shows the statistics of four ingredients used in the production of a type of cake from year 2016 to year 2020.



Ingredient	Price index in year 2020 based on year 2016	Weightage
	110	6
	160	3
	90	m
	120	2

The composite index for the cost of making the cakes in the year 2020 based on year 2016 is 116.

(*a*) Based on the information given, complete the table above.

(*b*) Given the butter costs RM 0.50 more than flour in year 2020.
Find the value of x.

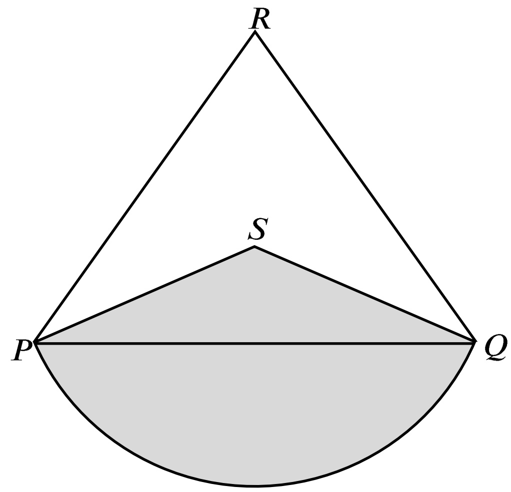
(*c*) Find the value of m.

(*d*) The production cost is expected to increase by 50% from the year 2020 to the year 2024.
Calculate the percentage of changes in production cost from the year 2016 to year 2024.

[10 *marks*]

CIRCULAR MEASURE

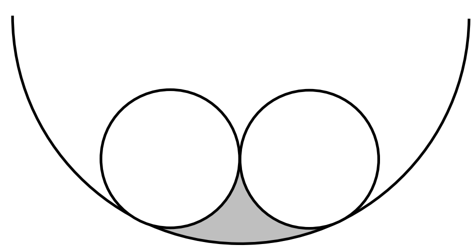
1 Diagram below shows a sector PSQ with centre S and an equilateral triangle PQR.



If length of PR is 12 cm. Show that the perimeter of tha shaded region is given by $8\sqrt{3}\,(1 + \frac{\pi}{3})$ cm.

[4 *marks*]

2 Diagram below shows two identical circles, each with a radius of 5 cm, touching each other.



It is given that both of the circles touches the arc of a larger circle that has a radius of 15 cm.

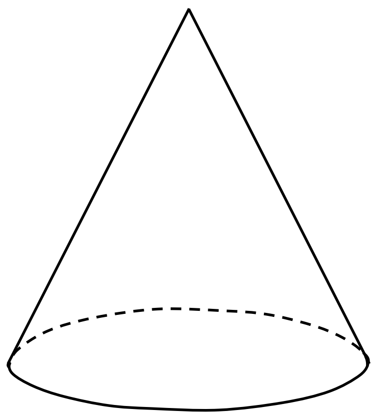
[Use $\pi = 3.142$]

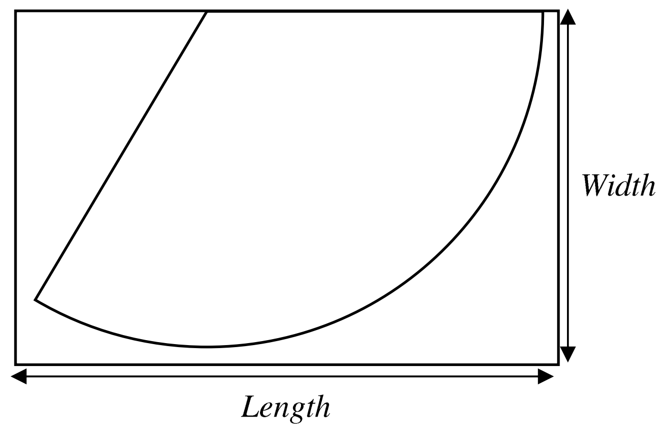
(*a*) Calculate the perimeter, in cm, of the shaded region.

(*b*) Find the area, in cm^2, of the shaded region.

[6 *marks*]

3 Susan wants to make a cap in the shape of a cone as shown in the diagram below. The height of the cone is 24 cm whereas the diameter of the cone is 17.8 cm. The other diagram shows the net of the cone drawn on a rectangular card.





[Use $\pi = 3.142$]

(*a*) Calculate the minimum length and width, in cm, of the rectangular card so that the cap can be made. Round off your answer to the nearest integer.

(*b*) Hence, by using your answer in (*a*). Calculate the area, in cm^2, of the rectangular card that is left unused.

[7 *marks*]

4 The Mathematics Society of SMK Muhibah organised a competition to design a logo for the society.

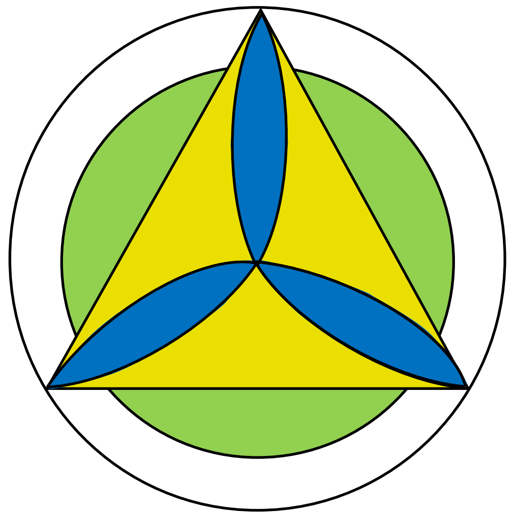


Diagram above shows the circular logo designed by Amar. The three blue regions are congruent and have a perimeter of 20π cm.

[Use $\pi = 3.142$]

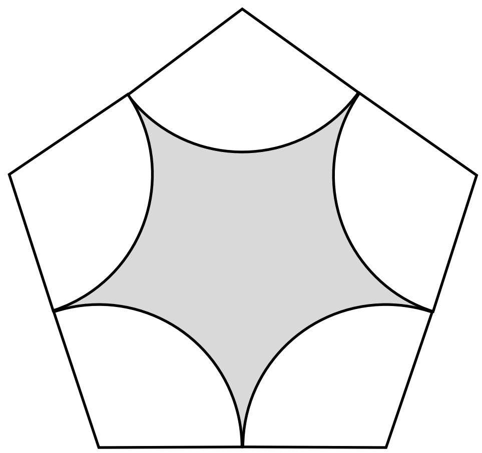
Find

(*a*) the radius, in cm, of the logo, to the nearest integer,

(*b*) the area, in cm^2, of the yellow coloured region.

[7 *marks*]

5 Diagram below shows a regular pentagon formed by five congruent sectors centered at the vertices of the pentagon. Each sector touches the adjacent sector.



It is given that the perimeter of the shaded region is 18π cm.

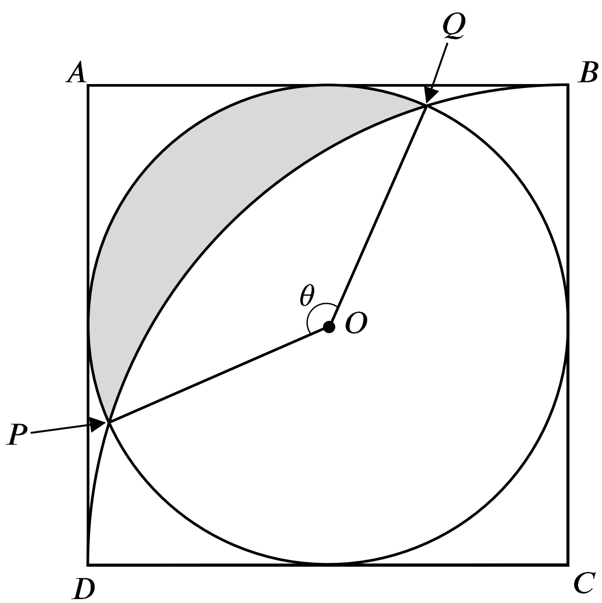
[Use $\pi = 3.142$]

(*a*) Calculate the side length, in cm, of the regular pentagon.

(*b*) A circle with maximum circumference is drawn in the shaded region.
Calculate the area, in cm^2, of the shaded region.

[8 *marks*]

6 Diagram below shows a circle with centre O inscribed in a square $ABCD$ with side length of 10 cm. Sector BCD is a quadrant of a circle with centre C. P and Q are two points located on arc BD.



It is given that PQ is a minor arc of the circle with centre O subtended by the angle θ.

[Use $\pi = 3.142$]

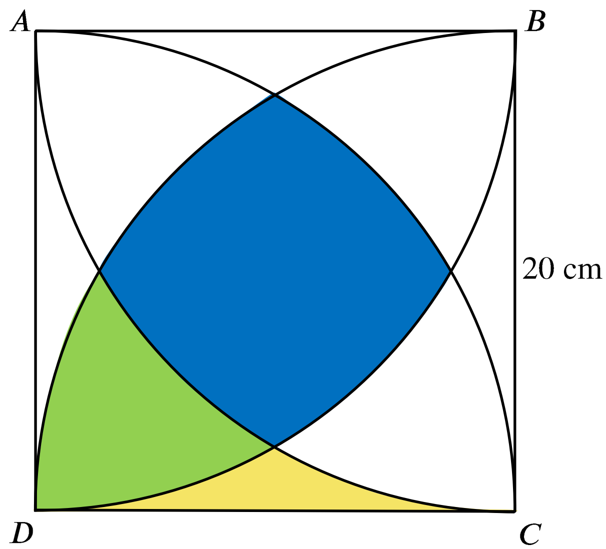
Find

(a) the value of θ, in radians,

(b) the area of the shaded region, in cm^2.

[10 *marks*]

7 Diagram below shows a square $ABCD$ formed by four congruent quadrants with radius of 20 cm centered at A, B, C and D.



[Use $\pi = 3.142$]

(a) Show that the area of the blue region is around 126.11 cm^2.

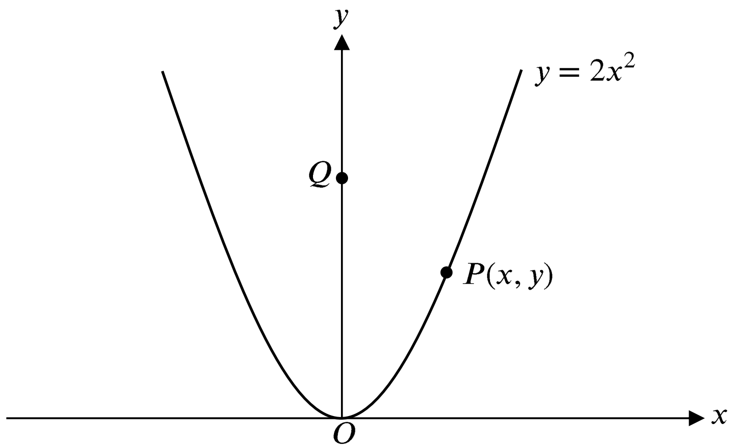
(b) Find the area, in cm^2, of the green region.

(c) Find the area, in cm^2, of the yellow region.

[10 *marks*]

DIFFERENTIATION

1 Diagram below shows a parabola shaped road with the equation $y = 2x^2$. O is the centre of the city with coordinates $(0, 0)$. Q is a landmark located on the y-axis that is 1.75 units away from point O as shown.



Point $P(x, y)$ represents a car travelling along the parabolic road. The distance between P and Q is given by the equation $s(x) = \sqrt{4x^4 - 6x^2 + \frac{49}{16}}$.

If 1 unit represents 1 km, determine the positions of P when it is the nearest to Q.

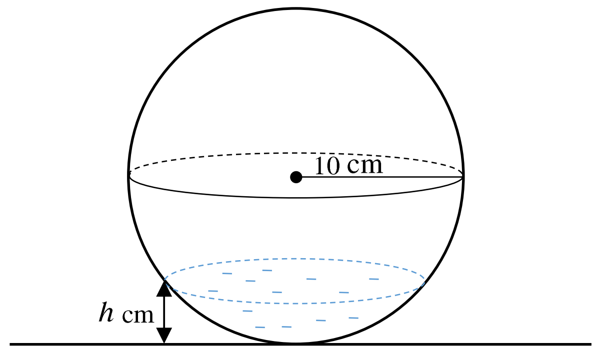
[4 *marks*]

2 A rectangular sheet of cardboard measuring 16 cm by 10 cm is to be formed into an open cuboid box by cutting identical squares from each corner and folding up the sides.

Determine the side length of the square so that the cuboid box has the maximum volume.

[3 *marks*]

3 Diagram below shows a spherical container with radius 10 cm being filled with water such that the water level, h cm, increases at a rate of 0.5 cm s^{-1}.



(a) Show that the area of the horizontal surface of the water, A cm^2, in the container, is represented by the equation $A = \pi(20h - h^2)$.

(b) Find the rate of increase of the horizontal surface of the water when the water level is at 6 cm.

(c) The volume of the water in the spherical container is V cm^3.
Find the range of $\frac{dV}{dt}$, where t is the time, in seconds, if water is leaking from the bottom of the container.

[6 *marks*]

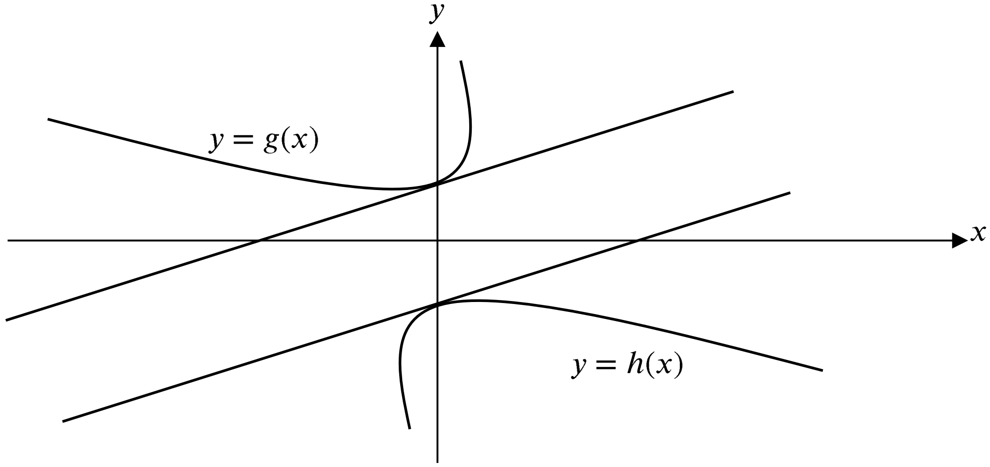
4 A curve is represented by the equation $y = x^3 + 3x^2 - 9x + 2$.

(a) Find the coordinates of the stationary points of the curve and determine the nature of each stationary point.

(b) Hence, determine the coordinate of the point where the rate of decrease of y is the greatest.

[7 *marks*]

5 Diagram below shows two curves $y = h(x)$ and $y = g(x)$. Two straight lines are drawn such that the lines are tangent to both curves when $x = 0$.

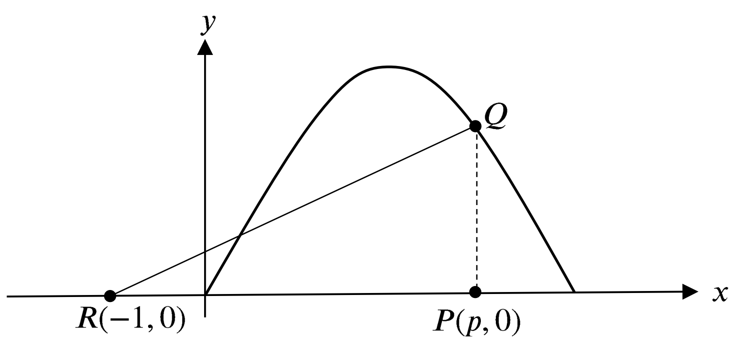


The two tangent lines are parallel and have a gradient of 1. The tangents to $g(x)$ and $h(x)$ has a y-intercept of c and $-c$ respectively.

If $f(x) = \dfrac{h(x)}{g(x)}$. Find $f'(0)$, in terms of c.

[4 *marks*]

6 Diagram below shows a part of the curve $y = 6x - x^2$ along with points $R(-1, 0)$ and $P(p, 0)$.



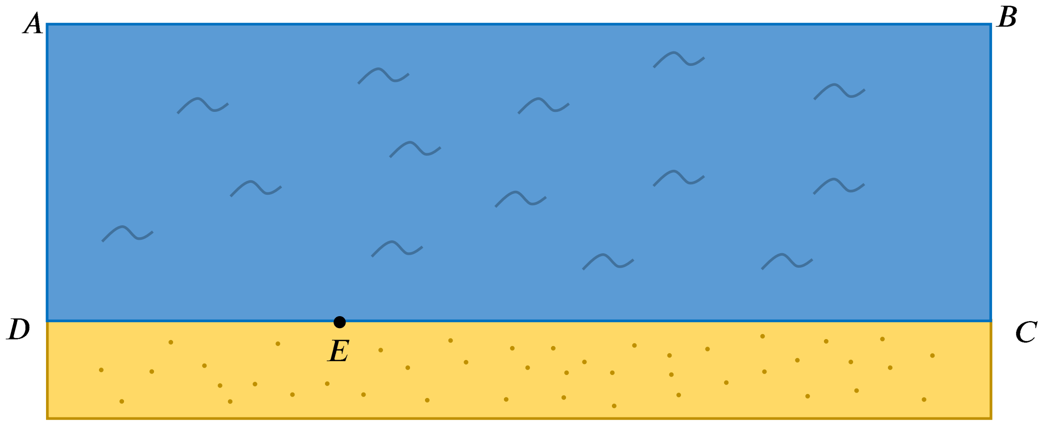
P is a moving point such that p increases at a rate of 10 units s^{-1} and PQ is always parallel to the y-axis.

(a) State the length, L, of QR in terms of p.

(b) Hence, calculate the rate of change of L when $p = 2$.

[6 *marks*]

7 Diagram below shows a sea $ABCD$ and a straight shore CD. Point E lies on line CD.

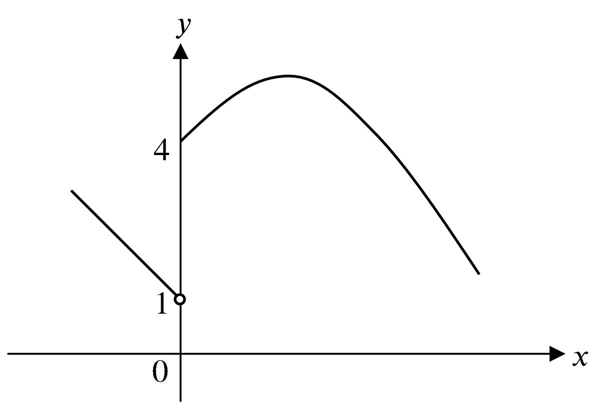


Mukhriz rows his canoe from point A to E where A is 30 m from the nearest point D, and E is x m from D. He then cycles from E to C where $CD = 400$ m.

If Mukhriz rows with a velocity of 40 m min^{-1} and cycles at 50 m min^{-1}. Find the distance from D to E.

[5 *marks*]

8 Diagram below shows a graph of part of a function $y = f(x)$.



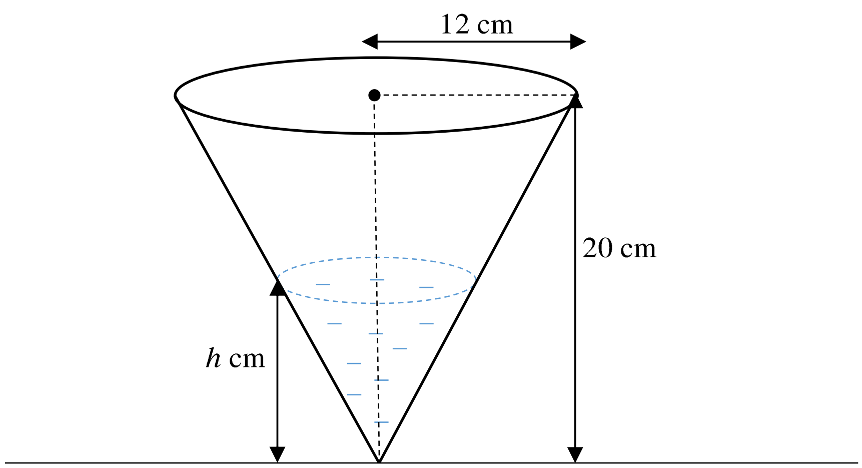
Based on the graph,

(*a*) find $f(0)$,

(*b*) determine whether $\lim_{x \to 0} f(x)$ exists, explain your answer.

[3 *marks*]

9 Diagram below shows an inverted cone with a base radius of 12 cm and a height of 20 cm being filled with water such that the water level increases at a rate of 2 cm s^{-1}.



Given the height of the water in the cone is h cm.

(*a*) Show that the volume of water, V cm^3, in the cone is $V = \frac{3}{25}\pi h^3$.

(*b*) Find the rate of change of volume of water in the cone when the water level is at 5 cm, in terms of π.

(*c*) i) Find the approximate change in the volume of water when the height, h increases from 5 cm to 5.01 cm, in terms of π.

ii) Show that an increase of $p\%$ in the height of the water causes an increase of $3p\%$ in its volume.

[10 *marks*]

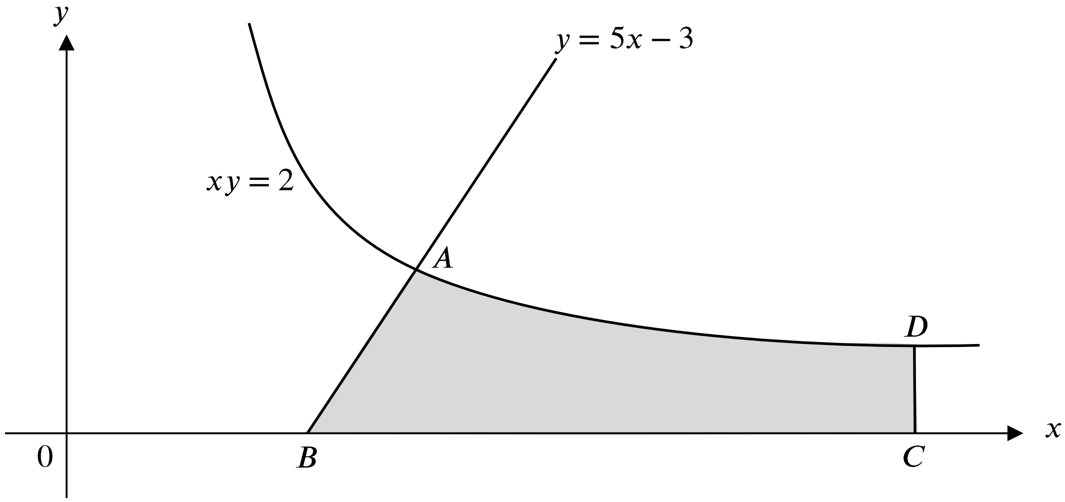
10 Water is being poured at a rate of 8 m^3 min^{-1} into an inverted conical shaped tank that is 20 m deep and 10 m in diameter at the circular base at the top. The tank has a leak at the tip, and the water level is rising at a constant rate of 50 cm min^{-1}.

Determine the rate of leaking of the water, in terms of π, when the water is 16 m deep.

[6 *marks*]

INTEGRATION

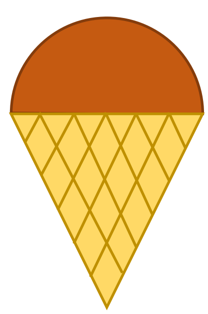
1 Diagram below shows part of a curve $xy = 2$ intersecting the straight line $y = 5x - 3$ at point A. The straight line meets the x-axis at point B. Point C lies on the x-axis and point D lies on the curve such that the line CD has the equation $x = 3$.



It is given that $\int a\frac{f'(x)}{f(x)}\,\mathrm{d}x = \ln x + c$. Find the exact area of the shaded region, give your answer in form of $p + \ln q$, where p and q are constants.

[6 *marks*]

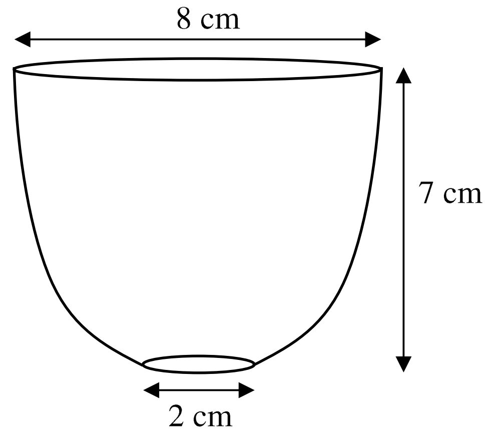
2 Diagram below shows an ice cream. The top surface of the ice cream can be represented by the equation $y = -\frac{1}{2}x^2 + 9$.



The height and radius of the ice cream cone are 7 cm and 2 cm respectively.
Given 1 unit represents 1 cm, calculate the volume, in terms of π, of the ice cream.

[4 *marks*]

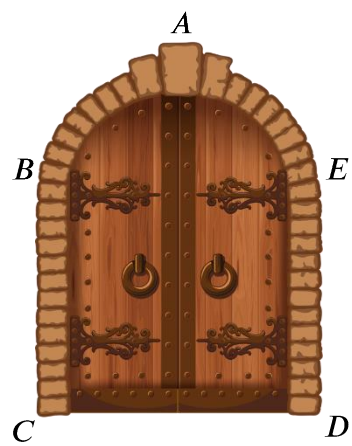
3 Diagram below shows a side elevation of the inner surface of a unique cup which can be represented by the equation $y = ax^3$, where a is a constant.



Given 1 unit represents 1 cm, calculate the volume, in cm^3, of the cup. Give your answer in terms of π.

[4 *marks*]

4 Diagram below shows the front view of a door.

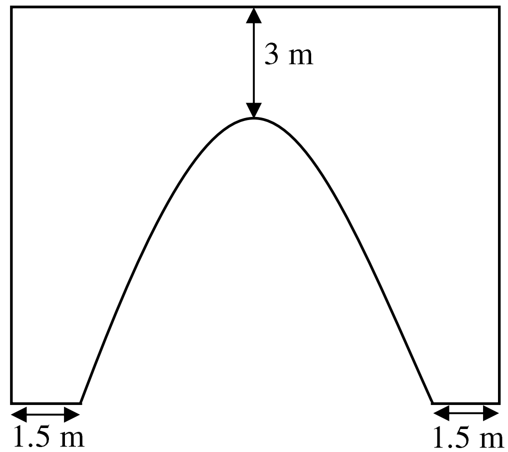


BC and ED are vertical straight lines whereas BE is a horizontal straight line. BAE is a symmetrical curve which is part of the graph $y = 4 + x - \frac{x^2}{4}$, such that the distance between A and the midpoint of straight line BE is 1 m. Given that $BC = ED = 6$ m, and A is the highest point from CD.

If 1 unit represents 1 m, calculate the surface area, in m^2, of the door.

[7 *marks*]

5 Diagram below shows the front view on an arch.

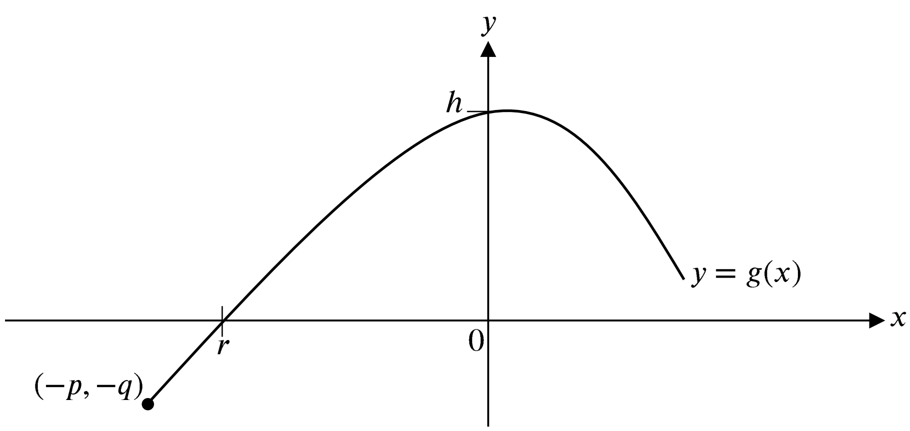


Company X is assigned to paint the front wall of the arch. The entrance of the arc is represented by the equation $y = ax^2 + c$, where a and c are constants. The width and the height of the wall is 7 m and 11m respectively. Paint is sold in a 5 litre bucket and 1 litre of paint can cover an area of 10 m^2 of a single coating.

Given 1 unit represents 1 metre, calculate the minimum number of bucket of paints needed to be bought if the company has to complete the painting with 3 coatings in total.

[7 *marks*]

6 Diagram below shows part of a curve $y = g(x)$.

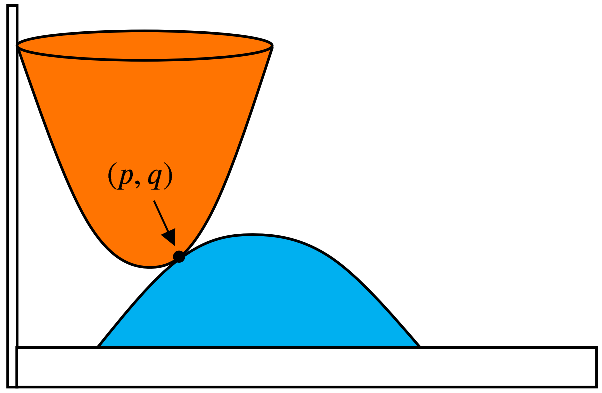


On the diagram, shade the area that is represented by the expression:

$$\int_{-q}^{h} -p \ dy + \int_{-p}^{r} g(x) \ dx - \int_{0}^{r} g(x) \ dx$$

[2 *marks*]

7 Diagram below shows two containers on a table beside a wall. Both containers touches each other at point (p, q). It is given that the gradient function of container A is $4 - 2x$ and the gradient function of container B is $hx - 8$, where h is a positive constant.



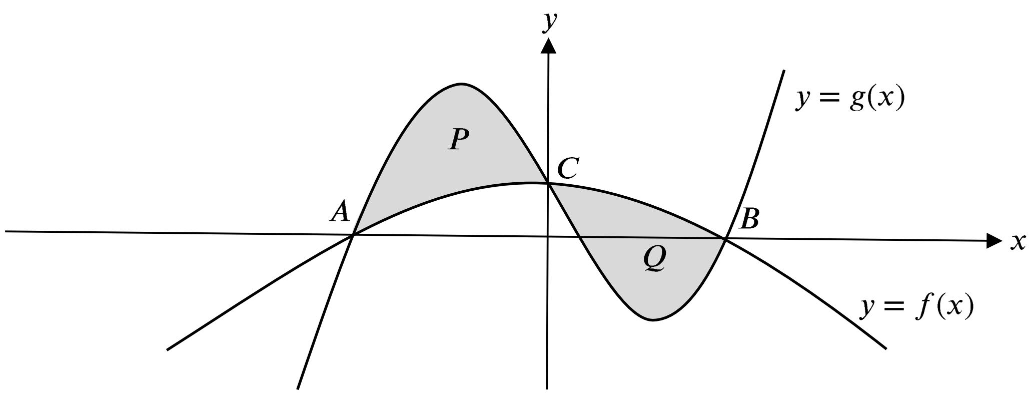
(*a*) Identify which coloured container is container A and container B.
Show your calculations to support your answer.

(*b*) Two lines, L_1 and L_2 are drawn such that L_1 is tangent to container A at (p, q), and L_2 is tangent to container B at (p, q).
Show that L_1 and L_2 are the same line.

(*c*) Hence, if $h = 6$ and $q = \frac{15}{4}$. Find the equation that represents container B.

[8 *marks*]

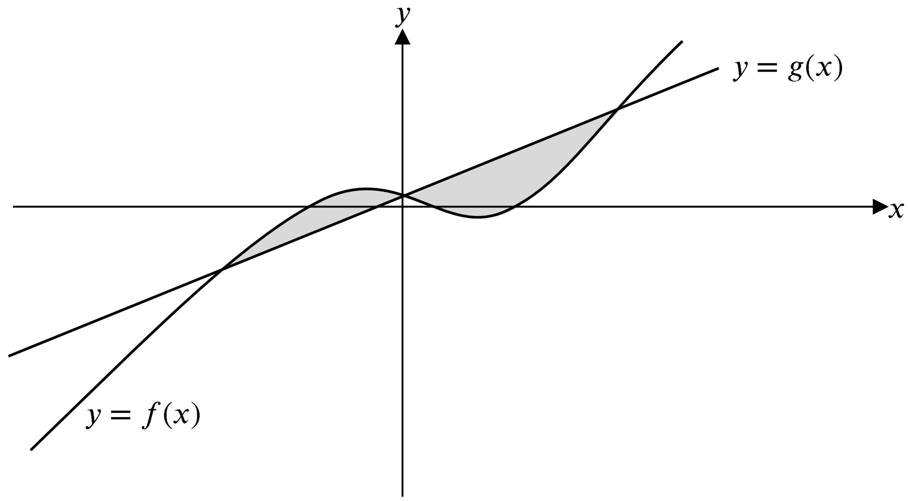
8 The graphs of $f(x) = -x^2 + 2$ and $g(x) = x^3 - x^2 - kx + 2$, the graphs intersect at the x-axis and y-axis at points A, B and C, creating two closed regions P and Q.



Without finding the value of k, show that these two regions have the same area.

[6 *marks*]

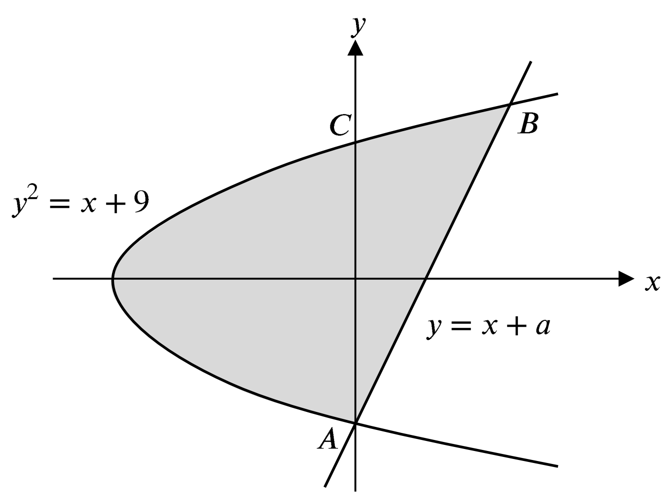
9 Diagram below shows the graphs of $f(x) = (x-1)(x^2-x-2)$ and $g(x) = 14x + 2$.



Find the exact area, in units^2, of the shaded area.

[7 *marks*]

10 Diagram below shows a straight line $y = x + a$ which intersects the curve $y^2 = x + 9$ at point A. It is given that the curve intersects the y-axis at points A and C as shown in the diagram.



(a) Find the value of a.

(b) Calculate the area of the shaded region, in units^2.

(c) The volume generated, in terms of π, when the region bounded by the curve and the y-axis is rotated through 180° about the x-axis.

[10 *marks*]

PERMUTATION AND COMBINATION

1 A five-letter code is formed from the word SIXTEEN. Find the number of ways the code can be arranged if

(*a*) all letters can only appear once in the code,

(*b*) the code must start with a vowel and end with a consonant.

[6 *marks*]

2 Find the number of ways in which the letters from the word CHOOSE can be arranged if

(*a*) there are no restrictions,

(*b*) no two consonants can be next to each other.

[5 *marks*]

3 Jee Kim was given five different car keys and five different types of cars, such that one key can only match to one car. He has to start each car with a key at random.

Find the number of ways such that

(*a*) at least one key goes to the wrong car,

(*b*) exactly one key goes to the wrong car, explain your answer.

[4 *marks*]

4 An entrance to a private school contains six questions in Part A and seven questions in Part B. Each candidate needs to answer 10 questions, of which at least four questions are from Part A.

Calculate the number of ways a candidate can answer these 10 questions.

[3 *marks*]

5 A local committee are to be selected from four couples. If the husband and his wife cannot serve in the same committee together, find the number of ways to select these committee members if the committee

(*a*) consists of three members,

(*b*) consists of four members.

[5 *marks*]

6 (*a*) Given ${}^6C_n > 1$, list all possible values of n.

(*b*) Given ${}^yC_m = {}^yC_n$, express y in terms of m and n.

[2 *marks*]

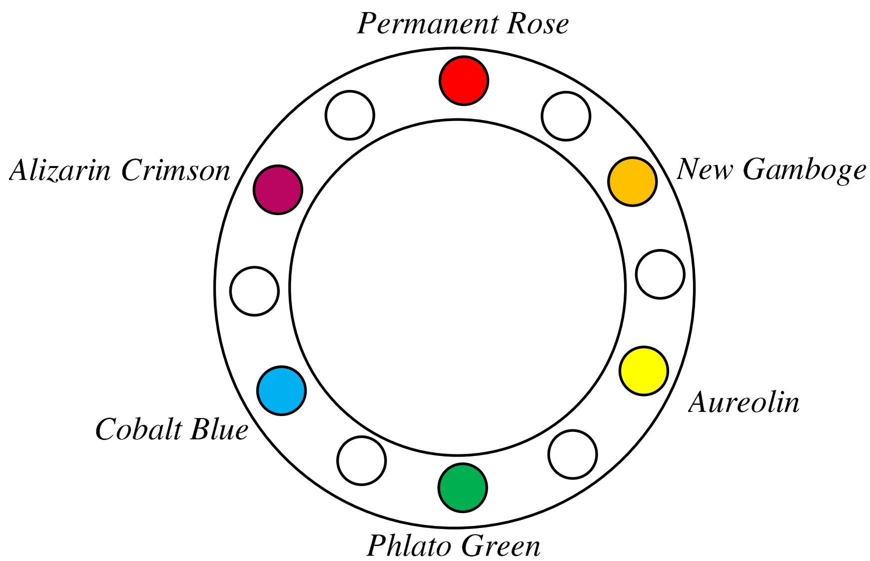
7 A puzzle-solving competition which consists of 16 contestants is held at SMK Merbau. For every round, each contestant meets one other contestant at a room to solve puzzles, if it is their first meeting, they shake hands with each other. Given that the competition consists of 10 rounds. Find the

(*a*) number of ways a handshake can occur, if three people already knew each other beforehand,

(*b*) maximum number of handshakes that can happen throughout the competition.

[4 *marks*]

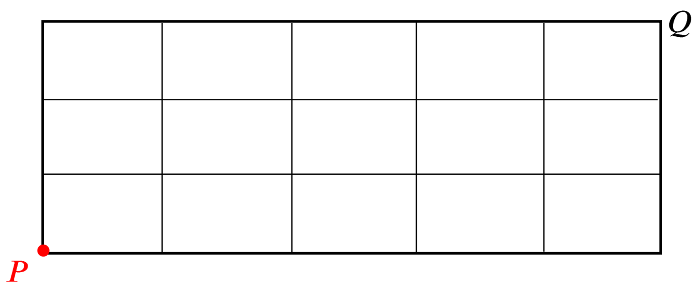
8 Diagram below shows a circular colour palette container used by a painter. The palette consists of 12 different spots to be filled in, 6 different colours has already been filled in.



Assuming the arrangement in the clockwise and anti-clockwise direction are different. Find the number of ways a painter can fill in the colour palette if Cobalt Blue and Aureolin colour must be side by side.

[3 *marks*]

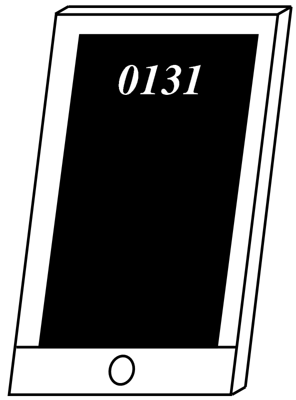
9 Diagram below shows a 5×3 grid and the position of particle P. The lines represent the path that can be taken by the particle.



Find the number of routes for the particle to move to point Q if the particle can only move upwards or rightwards.

[3 *marks*]

10 Diagram below shows a four-digit passcode '0131' set by San on his smartphone.



He wants to reset the passcode such that the new passcode cannot consists of the digit 1 followed by the digit 3.

Calculate the number of different passcodes that can be formed.

[3 *marks*]

11 Find the number ways to arrange all of the letters in the word ACETATE if only one of the letter A is next to the letter C.

[4 *marks*]

12 If ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, find ${}^{r}P_{2}$.

[3 *marks*]

13 If all permutations of the letters of the word 'AGAIN' are arranged in the order as in the dictionary. Find the 49^{th} word.

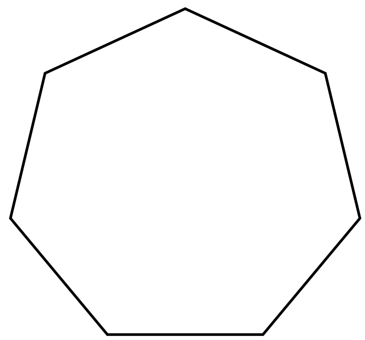
[4 *marks*]

14 Suppose m men and n women, where $m \geq n - 1$, are to be seated in a row such that no two women sit side by side. Show that the number of ways in which they can be seated is

$$\frac{m!\,(m+1)!}{(m-n+1)!}$$

[3 *marks*]

15 Diagram below shows a regular heptagon.



Find

(*a*) the number of triangles that can be formed from the vertices of the heptagon,

(*b*) the number of diagonals that can be drawn from the heptagon.

(*c*) the number of rectangles that can be formed from the vertices of the heptagon.

[4 *marks*]

16 Find the number of ways that four married couple can be seated at a round table if

(*a*) there are no restrictions,

(*b*) no two men sit next to each other,

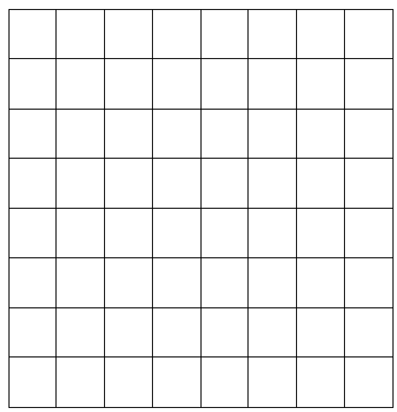
(*c*) the spouses are seated opposite to each other.

[5 *marks*]

17 The straight lines L_1, L_2 and L_3 are parallel and lie on the same plane. A total number of 10 points are taken on L_1; 7 points on L_2 and 5 points on L_3. Calculate the maximum number of triangles that can be formed with these points such that the points are the vertices of the triangle.

[3 *marks*]

18 Diagram below shows an 8×8 grid. It is given that each tile in the grid have the same size.



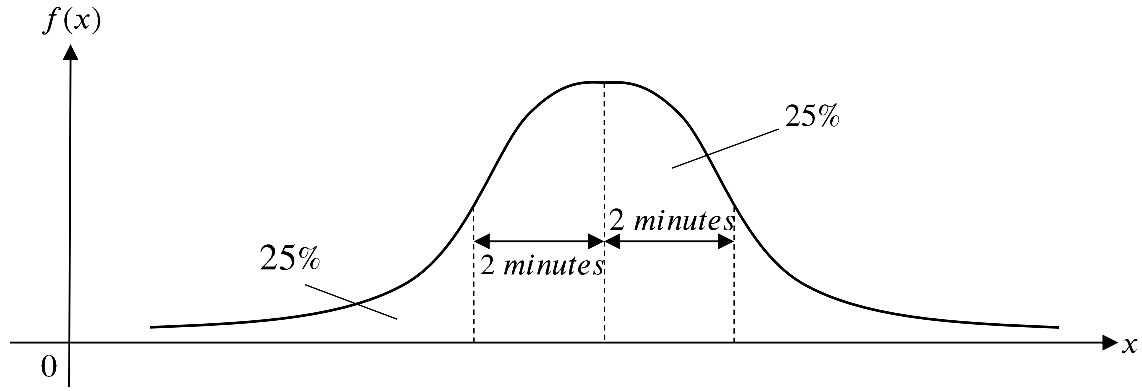
(*a*) Find the number of quadrilaterals that can be formed with the grid lines.

(*b*) Find the number of quadrilaterals that are not squares can be formed with the grid lines.

[5 *marks*]

PROBABILITY DISTRIBUTION

1 Diagram below shows the normal distribution graph of the time for a school bus to arrive at a school.



(a) Find the standard deviation.

(b) It is given that the mean time for the bus to arrive at the school is 7.15 a.m. Students are considered late if they arrived after 7.20 a.m. Lea takes the bus to the school.
Calculate the probability that Lea will be late. Give your answer correct to three significant figures.

[4 *marks*]

2 A crossword puzzle is published in a newspaper daily.
Sarah could complete, on average, 3 out of 5 crossword puzzles.

Calculate

(a) the probability that Sarah could complete at least 6 crossword puzzles in a week,

(b) the probability that in four certain weeks, Sarah could complete at least 6 crossword puzzles in the first week, and also in any two out of the next three weeks.

[5 *marks*]

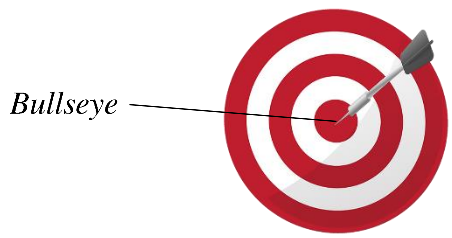
3 The heights of a group of Form 5 students are normally distributed with a mean of 165 cm and a standard deviation of 10 cm.

(*a*) Find the percentage of students whose height exceeds 180 cm.

(*b*) It is given that 25% of the students have heights between p cm and q cm, where $q > p$.
If the value of $q - p$ is as small as possible, find the value of p and of q.

[5 *marks*]

4 Diagram below shows a dart's target board at a dart game booth in a funfair.



The booth offers 3 darts per game. The customers have to pay RM 5 to play a game, a toy bear will be given to customers who are able to hit the bullseye for the three darts' thrown in a game. Bob is a dart player. By average, he hits the bullseye 7 times out of 10 darts thrown.

(*a*) Bob would play the game if he had at least 90% chance to win at least one toy bear by spending RM 30.
By mathematical calculation, suggest to Bob whether he should play the game or otherwise.

(*b*) Find the theoretical minimum number of games that Bob needed to play so that he can get 4 toy bears.

[6 *marks*]

5 An analysis is conducted on the SPM examination in a school.

(*a*) It is found that in the school, 150 of its 250 Form 5 students take the subject of Additional Mathematics.

i) If 8 students are chosen at random among the Form 5 students, find the probability that at least 2 students take the subject of Additional Mathematics.

ii) If the variance of students who sat for the subject of Additional Mathematics in the SPM examination is 33.36, find the number of students who probably have sat for the subject of Additional Mathematics in the SPM examination.

(*b*) The table below shows the average marks obtained by the students for the subject of Additional Mathmatics. The marks are graded as *A*, *B* and *C*.

Grade	A	B	C
Average marks, X	$X \geq m$	$60.0 \leq X \leq m$	$n \leq X \leq 60.0$

The average marks are normally distributed such that $X \sim N\,(59.7, 125.44)$. It is found that 10% of the students obtained Grade *A* and 15% of the students obtained Grade *C*.

Find the value of *m* and *n*.

[10 *marks*]

6 Ibrahim goes for fishing by bus, he only goes for fishing if he wakes up early in the morning and it didn't rain on that particular day. In any given week, he wakes up early for five days. The probability of raining in any given day is 60% and the probability of him missing the bus in any given day is $\frac{3}{8}$.

Find the probability that he did not go fishing on a certain day.

[3 *marks*]

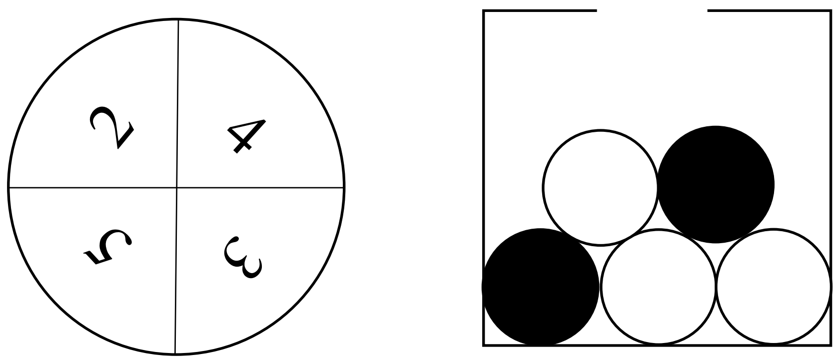
7 The probability that Jessy will win in a badminton competition is $\frac{1}{3}$. If she participates in n competitions, the probability for Jessy to win once in the badminton competition is 18 times the probability of losing in all of the competitions.

(*a*) Find the value of n.

(*b*) Find the expected number of games that Jessy won in the competition.

[5 *marks*]

8 Diagram below shows a spinning wheel which has 4 equal parts with different points used in a lucky draw in a supermarket. After spinning the wheel twice, the customers who obtain at least 9 points are given a chance to draw a ball randomly from a box as shown in another diagram. A prize will be given if a black ball is obtained.



(*a*) Find he probability that a customer is qualified to draw a ball.

(*b*) A customer who obtains ten points but fails to draw a black ball is given the privilege for the second draw without replacing the first ball drawn.
Find the probability that the customer will get the prize.

[4 *marks*]

9 Table below shows the number of white and green marbles of brand M and N.

Colour \ Brand	M	N
Green	3	4
White	2	6

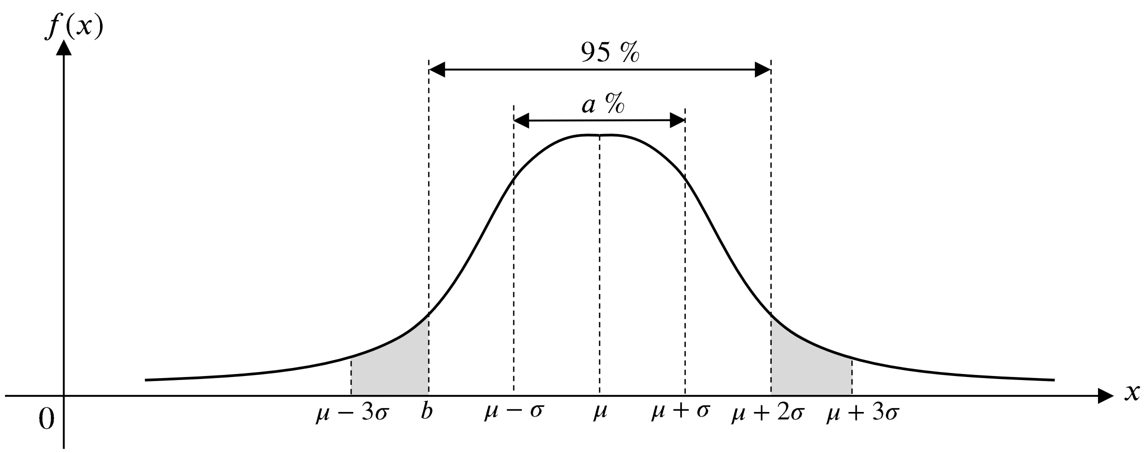
All of the marbles are put into a bag. Karen selected four marbles from the bag at random without replacement.

Calculate the probability that Karen selected one green marble or two brand M marbles.

[6 *marks*]

10 A continuous random variable X has a normal distribution, which is $X \sim N(\mu, \sigma^2)$.

Diagram below shows a normal distribution graph for the continuous random variable X.



It is given that the two shaded regions have the same area. Without using a calculator or referring to the table for the upper tail probability for the normal distribution $N(0, 1)$,

(*a*) express $P(|Z| > 1)$ in terms of a,

(*b*) express the possible values of k in terms of b and σ, if $P\left(Z > \left|\frac{k-\mu}{\sigma}\right|\right) = 0.025$,

(*c*) find the value of $P[X \leq (\mu - \sigma)]$ if $P(-2 < Z < 1) = 0.815$.

[5 *marks*]

TRIGONOMETRIC FUNCTIONS

1 (*a*) i) Without using the half-angle formulae, prove that $\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$.

ii) Hence, without using a calculator, find the value of $\tan\frac{\pi}{8}$.
State your answer in the form of $p - \sqrt{q}$, where p and q are constants.

(*b*) i) Sketch the graph of $y = -\frac{3}{2}\sin A$ for $0 \le A \le 2\pi$.

ii) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for the equation $(\cot\frac{A}{2})(1-\cos A) = -\frac{A}{2\pi}$

[10 *marks*]

2 It is given that $\cos 2\gamma = -\frac{7}{25}$, such that $180° \le 2\gamma \le 270°$.
Without using a calculator, find the value of

(*a*) $\sin\gamma$,

(*b*) $\cos\gamma$,

(*c*) $\tan\gamma$.

[5 *marks*]

3 Solve the equation $\frac{1-\cos 2\theta + \sin 2\theta}{1+\cos 2\theta} = 6$ for $0° \le \theta \le 360°$.

[5 *marks*]

4 Without using the half-angle formulae, prove that $\operatorname{cosec} x + \cot x = \cot\frac{x}{2}$.

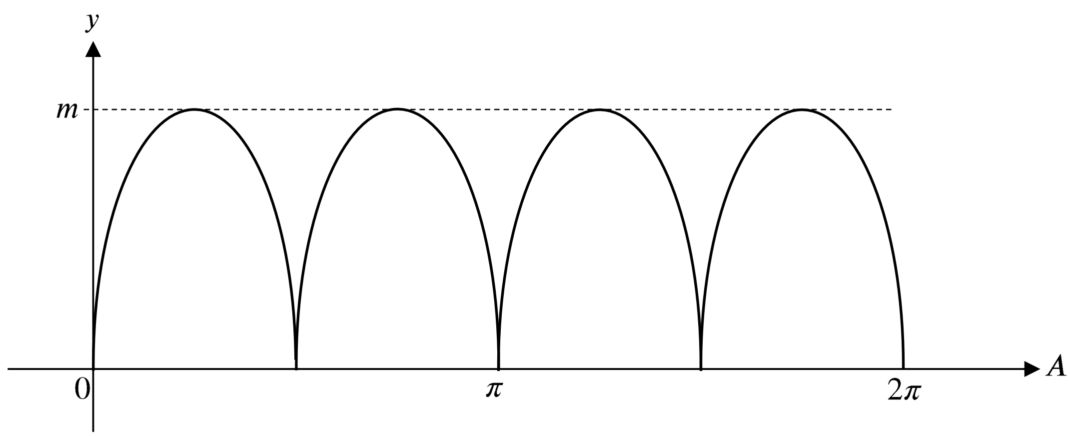
[4 *marks*]

5 Without using the half-angle formulae, prove the following identity.

$$\frac{\sin\alpha + \sin\beta - \sin(\alpha+\beta)}{\sin\alpha + \sin\beta + \sin(\alpha+\beta)} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

[6 *marks*]

6 Diagram below shows the graph of $y = |n \sin A \cos A|$ for $0 \leq A \leq 2\pi$.



(a) Express m in terms of n.

(b) There are 8 solutions when $y = k$, where k is a constant.
State the range of k in terms of m.

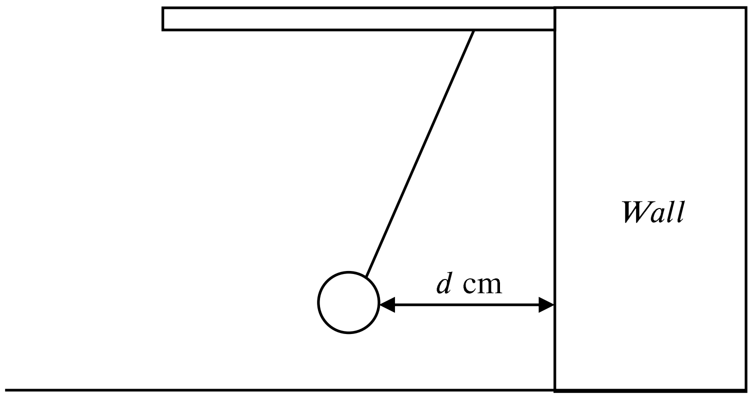
[3 *marks*]

7 Prove the following trigonometric identity:

$$\frac{\sin 2\theta + \cos 2\theta - 1}{\sin 2\theta + \cos 2\theta + 1} = \frac{\tan\theta}{\tan(45^\circ + \theta)}$$

[3 *marks*]

8 A pendulum is hanged from a fixed points as shown in the diagram below



The pendulum makes one complete swing every 6 seconds. As it swings, its distance from the wall d cm, is measured with respect to time, t, in seconds. When $t = 0$, the distance from the wall is maximum, that is 100 cm. The minimum distance of d is 60 cm.

Assuming that d is a sinusoidal function of t, in radians, write an equation that expresses d as a function of t.

[4 *marks*]

9 It is given that A, B and C are the interior angles of an acute-angled triangle.

Show that $\tan C = \dfrac{\tan A + \tan B}{\tan A \tan B - 1}$.

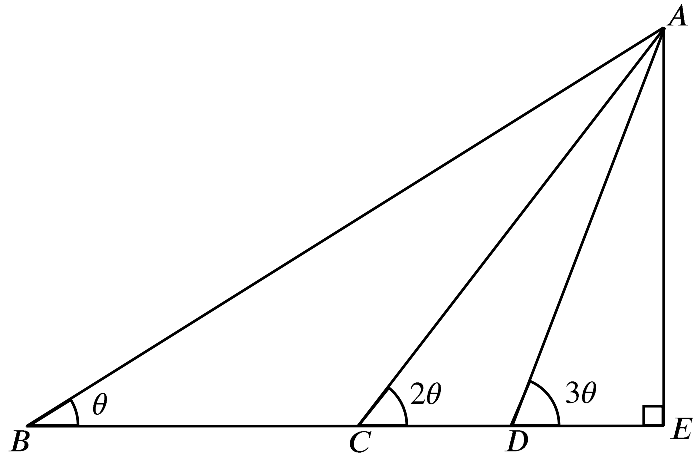
[3 *marks*]

10 (*a*) Without using half-angle formulae, prove that $\cot \dfrac{x}{2} = \sqrt{\dfrac{1+\cos x}{1-\cos x}}$.

(*b*) Hence, show that $\dfrac{1+\sin x+\cos x}{1+\sin x-\cos x} = \sqrt{\dfrac{1+\cos x}{1-\cos x}}$.

[6 *marks*]

11 In the diagram below, AE represents the height of a building. The angles of elevation of A from points B, C and D are $\theta, 2\theta$ and 3θ respectively.



The points B, C, D and E lie on a horizontal line. Given $BC = 11$ m and $CD = 5$ m. If $AE = h$ m and $DE = x$ m, find the height of the building in terms of x.

[4 *marks*]

12 (*a*) Show that $(\cos x + \sin x)^2 - (\cos x - \sin x)^2 = 2\sin 2x$

(*b*) i) Sketch the graph of $y = (\cos x + \sin x)^2 - (\cos x - \sin x)^2$ for $0 \leq x \leq 2\pi$.

ii) Hence, determine the range of number of solutions, n, for the equation $(\cos x + \sin x)^2 - (\cos x - \sin x)^2 = p$, where p is a constant and $-1 \leq p \leq 2$.

[6 *marks*]

13 (*a*) Show that $\dfrac{2\sin x}{\left(2\sin^2 x - 1\right)\sec x} = -\tan 2x$.

(*b*) i) Sketch the graph of $y = \dfrac{2\sin x}{(2\sin^2 x - 1)\sec x}$ for $0 \leq x \leq \pi$.

ii) Hence, determine the value of p, where p is a constant, such that the number of solutions for $\dfrac{2\sin x}{\left(2\sin^2 x - 1\right)\sec x} = p$ is not 2.

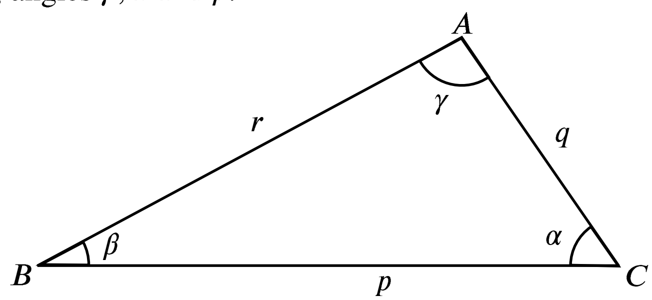
[6 *marks*]

14 (*a*) Sketch the graph of $y = 2\sin\left(x + \frac{\pi}{2}\right)$ for $0 \le x \le 2\pi$.

(*b*) Hence, using the same axes, sketch a suitable straight line to find the number of solutions for he equation $2\sin\left(x + \frac{\pi}{2}\right) + \frac{x}{\pi} = 2$ for $0 \le x \le 2\pi$.

[6 *marks*]

15 Diagram below shows a triangle PQR with sides p, q and r respectively and the corresponding angles γ, α and β.



Show that the area of the triangle, A, is given by the following formula:

$$A = \frac{p^2 \sin\alpha \sin\beta}{2\sin(\alpha + \beta)}$$

[4 *marks*]

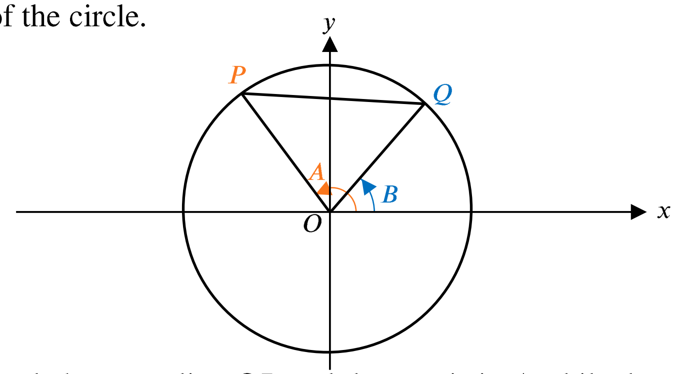
16 It is given that $q\sin\left(2\sqrt{p} + 2x\right) - 3 = \cos(\sqrt{p} + x)$, where $\tan(\sqrt{p} + x) = \frac{3}{4}$. Express $\sin\left(\sqrt{p} + x\right)$ in terms of q.

[4 *marks*]

17 Without using half-angle formulae, solve $\tan x = \dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}}$ for $0° \le x \le 360°$.

[3 *marks*]

18 Diagram below shows a unit circle with centre O $(0,0)$. The points P and Q lie on the cicrumference of the circle.

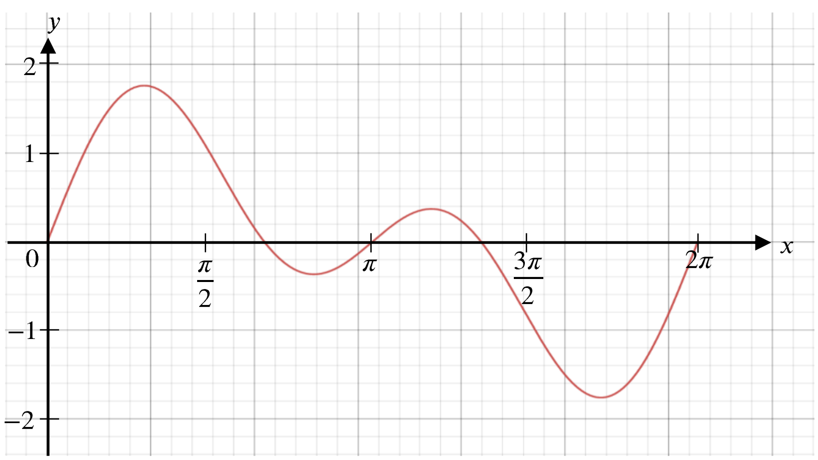


Given that the angle between line OP and the x-axis is A while the angle between line OQ and the x-axis is B.

By using two different methods, find the area of triangle POQ, in terms of A and B. Then, using the formulae obtained, show that $\sin(A - B) = \sin A \cos B - \cos A \sin B$.

[6 *marks*]

19 Diagram below shows the graph of $y = \sin 2x + \sin x$ for $0 \leq x \leq 2\pi$.



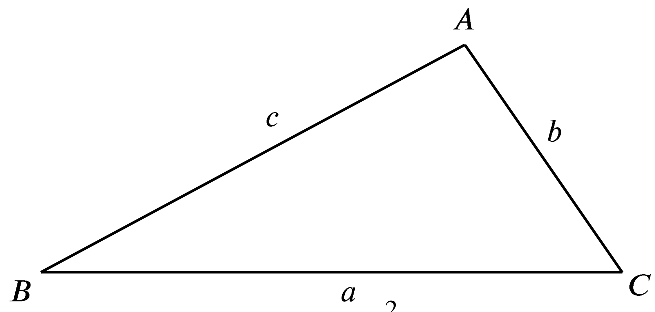
(a) Find the x-intercept of the graph.

(b) Using the same axes, sketch the graph of $y = \cos 2x + 1$. State the maximum value and the period of the graph.

(c) Hence, state the number of solutions to the equation $\sin 2x + \sin x = 2\cos^2 x$ for $0 \leq x \leq 2\pi$.

[7 *marks*]

20 Diagram below shows a traingle ABC with sides a, b and c their corresponding angle A, B and C.



It is given that the area of triangle ABC is $\dfrac{a^2}{3\sin A}$. Without using a calculator, find

(a) $\sin B \sin C$,

(b) the perimeter of triangle ABC, if $6\cos B\cos C = 1$ and $a = 3$.

[10 *marks*]

LINEAR PROGRAMMING

In this section, use a graph paper to answer the questions.

1 A factory produces two types of cupboards, namely cupboard *A* and cupboard *B*, Each cupboard require stwo types of raw materials *P* and *Q*. The amount of raw material needed to produce each unit of cupboard *A* and cupboard *B* are shown in the table below.

Cupboard	Number of raw materials	
	P	*Q*
A	2	3
B	5	2

The amount of raw materials *P* and *Q* available to the factory are 30 units and 24 units respectively. It is given that the number of cupcoard *A* produced is at most twice the number of cupboard *B*. Suppose the factory produces x units of cupboard *A* and y units of cupboard *B*.

(*a*) Write three linear inequalities, other than $x \geq 0$ and $y \geq 0$, which satifies all the contraints above.

(*b*) By using a scale of 2 cm to 2 units on the x-axis and 2 cm to 1 unit on the y-axis, construct and label the region *R* that satisfies all the above constraints.

(*c*) Based on the graph obtained in (*b*), find

i) the maximum number of cupboard *B* produced if the factory produces 4 units of cupboard *A*,

ii) the maximum profit
earned by the factory if the profit from one unit of cupboard *A* is RM 200 find one unit of cupboard *B* is RM 250.

[10 *marks*]

2 Seita Indah Secondary School will host a motivational camp. Participants of the camp are made up of x female pupils and y male students. The fee for a female pupil is RM 100 and the fee for a male pupil is RM 120. The number of pupils in the camp is based on the following constraints.

I The maximum number of pupils attending the camp is 80.

II The ratio of the number of female pupils to male pupils is at least 1 : 3.

III The total fees collected is not less than RM 5000.

(*a*) Write three linear inequalities that satisfy all the above constraints other than $x \geq 0$ and $y \geq 0$.

(*b*) Using a 2 cm scale for 10 pupils on the x-axis and y-axis, construct and label the region ***R*** that satisfies all the above constraints.

(*c*) Using the graph obtained in (*b*), find

i) the minimum number of male pupils if the ratio of the number of female to male pupils is 1 : 3,

ii) the maximum profit obtained if the school takes 25% of the total fees collected.

[10 *marks*]

3 A family in a village produces two types of rattan chairs, namely small rattan chairs and big rattan chairs. The family is able to get at least 60 kg of rattan a week as the raw material. A small rattan chair requires 3 kg of rattan while a big rattan chair requires 5 kg of rattan. There are 60 workers. Two workers are required to produce one small rattan chair while three workers are required to produce one big rattan chair.

(*a*) If x number of small rattan chairs and y number of big rattan chairs are produced in a week, write four linear inequalities that satisfy the above constraints.

(*b*) Using a scale of 2 cm to 5 rattan chairs on both axes, construct and label the region R that satisfies all the linear inequalities.

(*c*) The price for a small rattan chair is RM 40 and the price for a big rattan chair is RM 80.
From the graph obtained in (*b*), find the minimum and maximum income earned by the family.

[10 *marks*]

4 A paddy processing factory has x vans and y lorries to carry sacks of rice and flour to their customers. The factory has to send at least 80 sacks of rice and 96 sacks of flour to their customers. A van can carry 12 sacks of rice and 12 sacks of flour,whereas a lorry can carry 16 sacks of rice and 24 sacks of flour. It is given that the number of vans used is at most thrice the number of lorries used.

(*a*) Write three linear inequalities, other than $x \geq 0$ and $y \geq 0$, such that the three inequalities satisfy all the constraints above.

(*b*) Using a 2 cm scale to a vehicle on both axes, construct and label the region R that satisfies all the above constraints.

(*c*) From the graph obtained in (*b*), find

i) the range of the number of lorries if 4 vans are used.

ii) the maximum profit earned by the factory if the profit from one sack of rice is RM 10 and the profit from one sack of flour is RM 15.

[10 *marks*]

<u>KINEMATICS OF LINEAR MOTION</u>

In this section, solution by graph sketching is **not** accepted unless stated in the question.

1 A particle moves along a straight line such that its velocity, v m s^{-1}, is given by $v = t^3 - 4t^2 + 3t$, where t is the time, in seconds, after passing through fixed point O.
[Assume motion to the right is positive.]

Find

(*a*) the initial acceleration, in m s^{-2}, of the particle,

(*b*) the time interval, in seconds, when the acceleration of the particle is less than 6 m s^{-2},

(*c*) the time, in seconds, when the particle stop instantaneously,

(*d*) the total distance, in m, travelled by the particle until the particle returned to the fixed point O for the second time.

[10 *marks*]

2 A particle moves along a straight line and passes through a fixed point O. The velocity, v m s^{-1}, t seconds after passing through O is given by $v = t^2 - 6t + 8$. The particle stops instantaneously at points P and R.
[Assume motion to the right is positive.]

(*a*) Find the minimum velocity, in m s^{-1}, of the particle.

(*b*) Calculate the distance, in m, between point P and the point R.

(*c*) Sketch the velocity-time-graph for $0 \le t \le 7$. Then determine the range of values of t when the velocity of the particle is increasing.

[10 *marks*]

3 A particle moves along a straight line from a fixed point P. Its acceleration a m s^{-2}, at t seconds after leaving P is given by $a = mt + n$, where m and n are constants. The particle moves with an initial velocity of 30 m s^{-1}, while experiencing a deceleration 20 m s^{-2} and stops when $t = 2$.
[Assume motion to the right is positive.]

(*a*) Find the value of m and of n.

(*b*) Express the displacement function, s of the particle in terms of t.

(*c*) Find the value of t, in seconds, when the particle stops for the second time.

(*d*) Calculate the total distance, in m, of the particle travelled in the 2$^{\text{nd}}$ second.

[10 *marks*]

4 A particle moves along a straight line from an initial point. Its velocity, v m s^{-1}, at t seconds after passing through the initial point is given by $ht^2 + kt$ where h and k are constants. The particle stops instantaneously after 3 seconds with an acceleration at that time of 9 m s^{-2}.
[Assume the movement to the right is positive.]

Find

(*a*) the values of h and k,

(*b*) the time, in seconds, when the particle returns to the initial point,

(*c*) the acceleration, in m s^{-2}, when the particle returns to the initial point,

(*d*) the total distance, in m, travelled by the particle in the first 5 seconds.

[10 *marks*]

5 A particle moves along a straight line passing through point O. The particle's velocity is given by v m s^{-1}, such that $v = 2t^2 - 14t + 20$, where t is the time, in seconds, after passing through point O.
[Assume motion to the right is positive.]

Find

(*a*) the times when the particle is instantaneously at rest,

(*b*) the greatest speed of the particle in the interval $0 \le t \le 4$,

(*c*) the total distance travelled by the particle in the interval $0 \le t \le 4$.

[10 *marks*]

6 Two particles P and Q move along a straight line such that the displacement of the particles from fixed point O is s m. Particle P starts moving from A with $s_P = 10 + 8t - 8t^2$ and at the same time particle Q starts moving from B with $s_Q = 6t^2 - 9t - 12$, where t is the time, in seconds, after the two particles started moving.
[Assume rightwards as the positive direction.]

(*a*) Find the distance, in m, between A and B.

(*b*) Find the time, in seconds, when the particles met each other.

(*c*) Calculate the total distance, travelled by particle P when both particles met each other.

[10 *marks*]

7 Teacher Azizah conducted an experiment to determine the velocity of the trolley along a straight track. The velocity, v cm s^{-1}, of the troley after passing the fixed point O is given by $v = t^2 - 7t + 6$.

[Assume the movement to the right is positive]

(*a*) Calculate the initial velocity, in cm s^{-1}, of the trolley.

(*b*) Find the range of time, in seconds, when the trolley is moving to the left.

(*c*) Determine the range of time, in seconds, when the acceleration of the trolley is positive.

(*d*) Sketch the velocity-time graph for the movement of the trolley for $0 \leq t \leq 6$.

[10 *marks*]

8 A particle moves along a straight line and passes through a fixed point O with a velocity of -6 m s^{-1}. Its acceleration, a m s^{-2}, at t seconds after passing through O is given by $a = 8 - 4t$.

[Assume the movement in the rightwards direction as positive.]

(*a*) Find the maximum velocity, in m s^{-1}, of the particle.

(*b*) Find the time, in seconds, of the particle when it passed the fixed point O again.

(*c*) Sketch the velocity-time graph for the movement of the particle for $0 \leq t \leq 3$.

(*d*) Hence, find the total distance, in m, travelled by the particle in the first 3 seconds.

[10 *marks*]

FORMULA DERIVATION

1 Consider a quadratic equation $ax^2 + bx + c = 0$, where a, b and c are constants.

(*a*) By completing the square, show that $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(*b*) Hence, using the formula obtained, explain why only one real solution exists for the equation when $b^2 - 4ac = 0$ and no real solutions exist for the equation when $b^2 - 4ac < 0$.

[6 *marks*]

2 The definition of indices are as follows:

$$a^m = \underbrace{a \times a \times a \times \ldots \times a \times a}_{m \text{ times}}$$

By using the definition of indices, show that

(*a*) $a^m \times a^n = a^{m+n}$

(*b*) $a^m \div a^n = a^{m-n}$

(*c*) $(a^m)^n = a^{m \times n}$

(*d*) $(ab)^m = a^m b^m$

[6 *marks*]

3 It is given that dividing indices with the same base gives:

$$a^m \div a^n = a^{m-n}$$

By using the definition of indices and the law above, verify that

(*a*) $a^0 = 1$

(*b*) $a^{-m} = \dfrac{1}{a^m}$

[3 *marks*]

4 Suppose $a \geq 0$, then the definition of a root is as follow:

$$\boxed{\text{If } a^m = b, \text{then } a = \sqrt[m]{b}}$$

(*a*) Using the definition above, show that $\sqrt[m]{a^m} = \left(\sqrt[m]{a}\right)^m = a$.

(*b*) It is known that indices obey the following law:

$$\boxed{(a^m)^n = a^{m \times n}}$$

By using the law in (*a*) and the law above, prove that

i) $a^{\frac{1}{m}} = \sqrt[m]{a}$

ii) $a^{\frac{n}{m}} = \sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n$

[6 *marks*]

5 Recall that indices obey the following property:

$$\boxed{(ab)^m = a^m b^m}$$

(*a*) Use the property above to show that

i) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

ii) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$

(*b*) Suppose a fraction has a denominator $m\sqrt{a} + n\sqrt{b}$, where a, b, m and n are rational numbers. Explain why does the denominator always result in a rational number when multiplied with its conjugate surd. Show your calculations to support your answer.

[5 *marks*]

6 An incomplete definition of logarithm is given as follows:

$$\boxed{\text{If } a^x = N, \text{then} \log_a N = x}$$

By using laws of indices and the definition above, verify that

(*a*) $\log_a 1 = 0$

(*b*) $\log_a a = 1$

(*c*) $\log_0 a$ and $\log_1 a$ is undefined

(*d*) $\log_a 0$ is undefined

[7 *marks*]

7 It is given that logarithms can be defined as follows:

$$\boxed{\text{If } a^x = N, \text{then} \log_a N = x, \text{where } a > 0, a \neq 1 \text{ and } N > 0}$$

Suppose $a^p = m$ and $a^q = n$. By using laws of indices and the definition of logarithm, show that

(*a*) $\log_a m + \log_a n = \log_a mn$

(*b*) $\log_a m - \log_a n = \log_a \frac{m}{n}$

(*c*) $\log_a m^x = x \log_a m$

[5 *marks*]

8 The power law of logarithms is given as follow:

$$\boxed{\log_a m^n = n \log_a m}$$

By using the law above, as well as the definition of logarithm.

(*a*) Show that $a^{\log_a x} = x$

(*b*) Suppose $a^x = b$, show that $\log_a b = \frac{\log_c b}{\log_c a}$

[4 *marks*]

9 Suppose the first n terms of a progression is given by $T_1, T_2, T_3, \dots, T_{n-1}, T_n$, where T_n is the n^{th} term of the prorgession. Then the sum of the first n terms, S_n, of the progression is given by:

$$\boxed{S_n = T_1 + T_2 + T_3 + \cdots + T_{n-1} + T_n}$$

(a) Consider the progression to be an arithmetic progression with the first term as a and the common difference as d. Show that

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

(b) Consider the progression to be a geometric progression with the first term as a and the common ratio as r, where $r \neq 1$. Show that

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$$

[7 *marks*]

10 Consider a geometric progression with the first term as a and the common ratio as r, where $|r| < 1$. Then the sum of the first n terms is given by the formula:

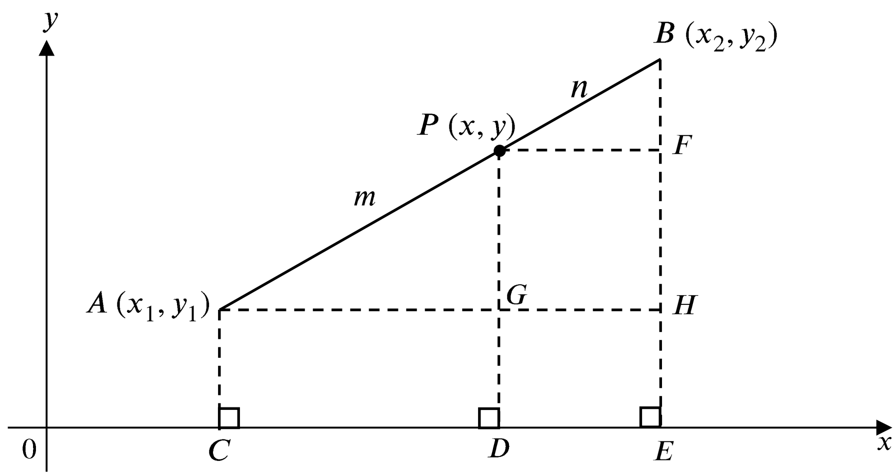
$$\boxed{S_n = \frac{a(1 - r^n)}{1 - r}}$$

By applying limits into the formula above, prove that the sum to infinity of the geometric progression, S_∞, is

$$S_\infty = \frac{a}{1 - r}$$

[2 *marks*]

11 In the diagram below, the coordinates points A and B are (x_1, y_1) and (x_2, y_2). $P\ (x, y)$ is a point which divides line segment AB such that $AP : PB = m : n$.

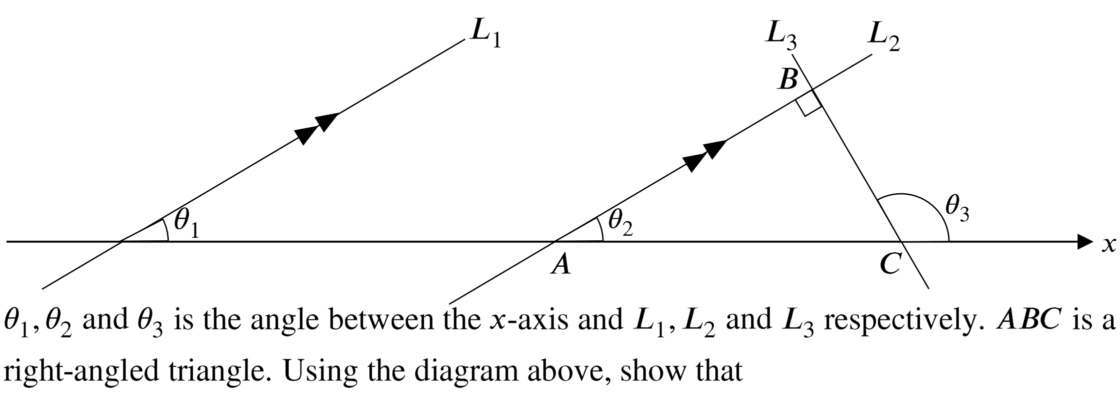


It is given that AC, PD and BE are perpendicular to the x-axis. Points F and G lie on the line BE and PD respectively. Using the diagram above, show that the coordinates of P is given by

$$(x, y) = \frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n}$$

[4 *marks*]

12 Diagram below shows three lines L_1, L_2 and L_3 with their respective gradients m_1, m_2 and m_3. It is given that L_1 is parallel to L_2 while L_2 is perpendicular to L_3.



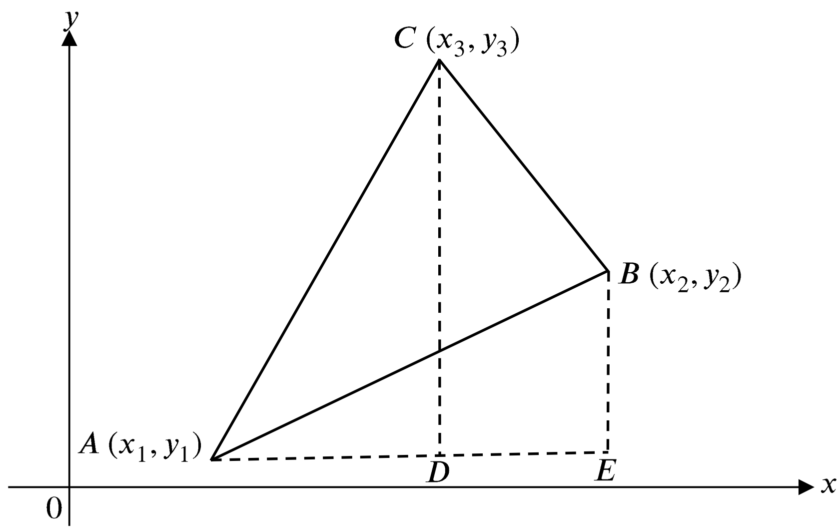
θ_1, θ_2 and θ_3 is the angle between the x-axis and L_1, L_2 and L_3 respectively. ABC is a right-angled triangle. Using the diagram above, show that

(*a*) $m_1 = m_2$

(*b*) $m_2 m_3 = -1$

[3 *marks*]

13 Diagram below shows a triangle with vertices $A\ (x_1, y_1)$, $B\ (x_2, y_2)$ and $C\ (x_3, y_3)$ on a Cartesian Plane.



The line CD and BE are parallel to the y-axis while line AE is parallel to the x-axis.

(*a*) Using the diagram above, show that the area of the triangle is given by:

$$\text{Area} = \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)]$$

(*b*) Predict what will happen to the area obtained from the formula in (*a*) when C is moved to a position lower than points A and B.

(*c*) Hence, modify the formula in (*a*) to prevent the situation in (*b*) from occuring.

[6 *marks*]

14 The area of a triangle with known coordinates of the vertices is given by the formula:

$$\boxed{\text{Area} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right|}$$

By using the formula above.

(a) Show that the area of a quadrilateral with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) is given by

$$\text{Area} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4)\right|$$

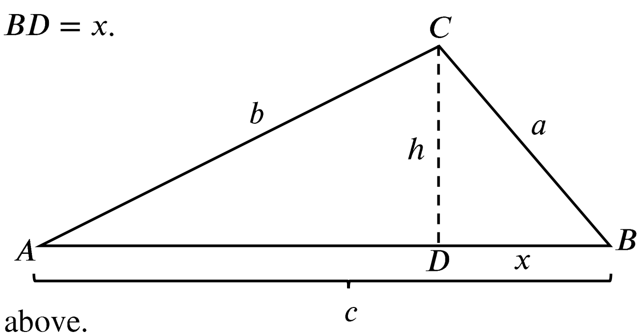
(b) Show that the area of a pentagon with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$ and (x_5, y_5) is given by

$$\text{Area} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_4 + x_4y_5 + x_5y_1) - (x_2y_1 + x_3y_2 + x_4y_3 + x_5y_4 + x_1y_5)\right|$$

(c) Using the formula given, as well as the formulae obtained in (a) and (b), state a generalised formula that is used to find the area of a polygon with vertices at $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$.

[7 *marks*]

15 Consider the triangle ABC in the diagram. The vertices A, B and C have their corresponding sides as a, b and c. D is a point on line AB such that CD is the height of the triangle, h. Given $BD = x$.



By using the diagram above.

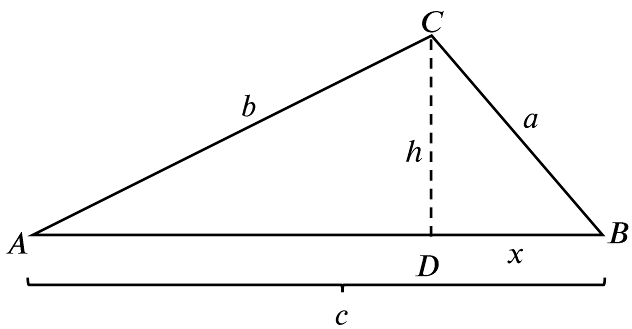
(a) Verify the sine rule.

(b) Prove the cosine rule.

(c) Show that the area of the triangle is half of the product between two of the sides of the triangle and the sine of the included angle.

[8 *marks*]

16 Consider the triangle ABC in the diagram. The vertices A, B and C have their corresponding sides as a, b and c. D is a point on line AB such that CD is the height of the triangle, h. Given $BD = x$. The interior angles of the triangle are not known.



By using suitable method. Show that the area of the triangle is given by

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s is the semiperimeter of the triangle, given by

$$s = \frac{a+b+c}{2}$$

[10 *marks*]

17 Recall that

π is an irrational constant defined as the ratio of the circumference of a circle to its diameter.

One radian is the measure of an angle subtended at the centre of a circle by an arc whose length is the same as the radius of the circle.

A circle has a radius r. Using the two definitions above.

(a) Find the circumference of the circle, in terms of π and r.

(b) The arc length of the circle subtended at the centre with angle 1 rad.

(c) The angle subtended at the centre of the circle by the arc with length same as the circumference of the circle.

(d) Hence, by using (c), find the value of π radians, in degrees.

[5 *marks*]